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Comparison of Robust Control Techniques for Use in Continuous Stirred Tank Reactor Control

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Abstract: This work deals with the design and the application of a robust control on a chemical reactor. The reactor is exothermic one. There are two parameters with only approximately known values in the reactor. These parameters are the reaction enthalpies. Three robust control techniques are used – robust PI controller found by plotting the stability boundary locus in the plane of controller parameters that is called (k_p, k_i)-plane, $\mathcal{H}_\infty$ control and μ-synthesis control. The presented simulation results confirm applicability of mentioned approaches to safe control of nonlinear processes.

Keywords: chemical reactor, robust control, Kharitonov system, $\mathcal{H}_\infty$ controller, DK-iteration.

1. INTRODUCTION

The continuous-time stirred reactors (CSTRs) are one of the most important equipment in chemical industry. It is well known that control of chemical reactors represents very complex problem (Luyben 2007). The process parameters (e.g. reaction rates, heat transfer coefficients, reaction rate constants) are not exactly known; this introduces uncertainty in the models of reactors (Laššák, 2010; Bakošová, 2012). CSTRs are complicated processes because of their nonlinearity (Kvasnica et al., 2010), potential safety problems (Ball and Gray, 2013), the possibility to have multiple steady states, and other.

Effective control of CSTRs requires application of some of advanced methods, as e. g. robust control (Gerhard et al. 2004; Tlacuahuac et al. 2005). Robust control has grown as one of the most important areas in the modern control design since works by (Zames 1983; Doyle 1984) and many others. The main task of robust control is to ensure the stability of the feedback control loop and to reach required control performance in the presence of uncertainty in the controlled system (Dulău, 2009).

The paper Matušů (2011) focuses on robust stabilization where the suitable parameters of a simple PI controller are determined through a combination of the Kronecker summation method, sixteen plant theorem, and an algebraic approach to control design in the ring of proper and stable rational functions. In Závacká et al. (2014), a robust PI controller is designed for stabilization of a reactor into the unstable steady state.

There exist various solutions also of the standard $\mathcal{H}_\infty$ problem. If there are uncertainties in the system model, some quantity combining the $\mathcal{H}_2$ norm and the $\mathcal{H}_\infty$ norm can be a measure of a system's robust performance. The same approach is used for convex parameterization of fixed-order $\mathcal{H}_\infty$ controllers in Yang et al. (2007). Many problems in systems and control are suitable for convex reformulation via LMIs (Scherer, 2006).

Very popular method for robust controller design is the DK-iteration. Since DK-iteration is an ad hoc approach applied to a nonconvex problem, the resulting controller can be suboptimal. In spite of the fact that the DK-iteration does not guarantee the convergence to a global optimum, it has been applied to a large number of academic case studies such as high purity distillation columns, CSTRs, and packed bed reactors (Morari and Zafiriou, 1989; Skogestad and Postlethwaite, 1996). For the dynamic system with time varying characteristic and parametric uncertainties, a sliding mode controller is developed and an optimal $\mathcal{H}_\infty$ controller is designed based on μ-synthesis with DK-iteration algorithm in (Moradi, 2012).

In this paper three approaches used for control of a continuous stirred tank reactor are compared. The first approach deals with the design of a robust PI controller for stabilization of the reactor with uncertain interval parameters. The robust PI controller parameters are found using the method that combines the sixteen plant theorem and plotting the closed-loop stability boundaries in the (k_p, k_i)-plane for sixteen Kharitonov plants. The robust stability region is the intersection of sixteen stable regions. The second approach deals with the design of $\mathcal{H}_\infty$ control technique, and the last approach deals with the design of robust control based on the μ-synthesis with DK-iteration. Simulations verify the robustness of designed controllers.

The paper is organized as follows. In Section 2, three methods for robust controller' design are described. In Section 3, the model of the CSTR is established. In Section 4, the controllers developed for the CSTR and the simulation

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results are presented, and finally, Section 5 offers some conclusion remarks.

2. ROBUST CONTROLLER DESIGN

2.1 *The Kharitonov systems*

The method for synthesis of robust PI controllers is based on plotting the stability boundary locus in the (k_p, k_i)-plane (Yeroglu and Tan 2008) and the subsequent choice of a stabilizing PI controller using the pole-placement method so that the prescribed behaviour of the closed-loop is achieved.

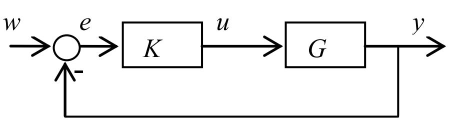

Fig. 1. Control system

Consider the closed loop as shown in Fig. 1. The block G represents the controlled process described by the transfer function (1), K is the PI controller (2), w is the set point, e is the control error, u is the control input and y is the controlled output.

$$G(s) = \frac{N(s)}{D(s)} \tag{1}$$

$$K(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \tag{2}$$

Decomposing the numerator and the denominator polynomials of (1) into their even and odd parts, and substituting $s = j\omega$, gives

$$G(j\omega) = \frac{N_e(-\omega^2) + j\omega N_o(-\omega^2)}{D_e(-\omega^2) + j\omega D_o(-\omega^2)} \tag{3}$$

The closed loop characteristic polynomial can be written as

$$\begin{aligned}\Delta(j\omega) &= [k_i N_e(-\omega^2) - k_p\omega^2 N_o(-\omega^2) - \omega^2 D_o(-\omega^2)] + \\ &+ j\omega[k_p N_e(-\omega^2) + k_i N_o(-\omega^2) + D_e(-\omega^2)]\end{aligned} \tag{4}$$

Then, equating the real and imaginary parts of $\Delta(j\omega)$ to zero, one obtains

$$k_p(-\omega^2 N_o(-\omega^2)) + k_i N_e(-\omega^2) = \omega^2 D_o(-\omega^2) \tag{5}$$

$$k_p N_e(-\omega^2) + k_i N_o(-\omega^2) = -D_e(-\omega^2) \tag{6}$$

The parameters of PI controller are calculated as a solution of (5) and (6) for given frequency ω.

2.2 *$\mathcal{H}_\infty$ Control*

Various techniques are available for the design of the $\mathcal{H}_\infty$-controller. It is known that the robust controller is designed to minimize the $\mathcal{H}_\infty$-norm of the plant. Three weight functions are added to the control system for loop shaping (Bansal and Sharma, 2013). The classical feedback control system structure with weighting is shown in Fig. 2.

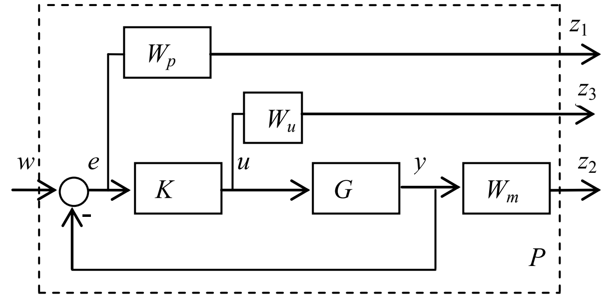

Fig. 2. Control system for the synthesis of $\mathcal{H}_\infty$ controller

For a generalized plant P following equations can be written:

$$\begin{aligned} z_1 &= W_p(w - Gu) \\ z_2 &= W_p Gu \\ z_3 &= W_u u \\ e &= (w - Gu)\end{aligned} \rightarrow \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ e \end{pmatrix} = \underbrace{\begin{pmatrix} W_p & -W_p G \\ 0 & W_m G \\ 0 & W_u \\ 1 & -G \end{pmatrix}}_{G_n} \begin{pmatrix} w \\ u \end{pmatrix} \tag{7}$$

$$\|P(G,K)\|_\infty = \left\| \begin{matrix} W_p S \\ W_m T \\ W_u KS \end{matrix} \right\|_\infty \tag{8}$$

S and T are the sensitivity and the complementary sensitivity functions, respectively, W_p is the performance weighting function which limits the magnitude of the sensitivity function and W_m is the robustness weighting function used to limit the magnitude of the complementary sensitivity function.

$$\begin{aligned} S(s) &= (I + G(s)K(s))^{-1} \\ T(s) &= (I + G(s)K(s))^{-1} G(s)K(s) \end{aligned} \tag{9}$$

$$\|S(j\omega)\| < \frac{1}{W_p(j\omega)}, \ \|T(j\omega)\| < \frac{1}{W_m(j\omega)} \tag{10}$$

A stabilizing controller $K(s)$ can be achieved by solving the algebraic Riccati equations, minimizing the cost function γ. $|P|=\gamma$ so, min$|P|\leq 1$.

2.3 *μ-synthesis with DK-iteration*

There is no analytical method to calculate a μ-optimal controller. However, a numerical method for complex perturbations known as *DK*-iteration can be used (Balas et al., 1998). The generalized open-loop representation of the configuration in Fig. 3 can be given by (11) (Griffin and Fleming, 2003). The input and output vectors for this configuration contain the inputs and outputs relating to the input uncertainty perturbation u_Δ and y_Δ. Uncertainty at the