Measurement 94 (2016) 806–811

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

# Measurement and mathematical modelling of heat loss in the pipe systems of a central heat distribution network

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ARTICLE INFO

*Article history:*
Received 4 September 2015
Received in revised form 26 August 2016
Accepted 16 September 2016
Available online 17 September 2016

*Keywords:*
Heat loss
Measurement
Dimensional analysis
Modelling

ABSTRACT

The measurement of physical parameters conducted on a central heat distribution network was used to determine heat loss using pipe dimensional analysis. All the developed models include all relevant physical parameters which are expected to have an impact on heat loss. The interdependence of these parameters is expressed as a function of similarity criteria. When creating a mathematical model, the so-called Buckingham $\pi$ theorem is used. The model consists of the dependency of only two parameters, which greatly simplifies the currently-used procedure of calculation based on heat transfer theory. The proposed way of expressing heat loss is further interpreted in the article by two simpler modifications. In the simplest variant, the interdependence of dimensionless criteria is subtracted from the original diagram. The results presented in this article are valid for a diameter 125 mm (DN125) overhead network with insulation. The scope of the model was validated for ambient temperatures of −20 to +30 °C and a transported water temperature from 40 to 70 °C.



## 1. Introduction

The determination of heat loss from a pipework system is a matter of serious concern for distributors of heat and hot water. At present, dealing with such losses is based on the heat transfer theory [1–3].

Based on detailed analysis, the analytical procedure formulating the heat losses is accompanied by complex calculation of the linear specific heat resistance of a given heat network. This resistance depends on several factors: the external temperature of the conducted material, ambient temperature, the quality of insulation thickness, the material and the nominal diameter of the pipe. Moreover, in order to formulate the amount of heat losses, it is essential to identify the heat transfer coefficient of the running water and that of the environment in which the supply is conducted. The heat transfer coefficient inside the transfer network depends mainly on the temperature of the medium, but also depends on the nature of the flow [4–7]. The external heat exchange coefficient consists of two parts that are linked to convection (forced or free) as well as to radiation. If the distribution is led via a free environment (unrestricted area), there are equations for determining both parts of the given coefficient [8]. If the distribution is conducted underground (through pipelines or without), there is no available precise method of expressing the coefficient of heat transfer to the surroundings. In this case, the heat loss of the distribution network could be obtained, for example, by the balance method [9,10]. The disadvantage of this method is that in very short networks equipped with quality thermal insulation, it is not possible, with the existing measurement technique installed on the network, to *obtain* the correct information on the temperature gradient with the required accuracy [10]. Moreover, older networks completely lack permanently installed equipment. The previously mentioned method requires one to know the flow rate of the distribution network at the time the temperature gradient is measured, which represents a further complication.

The calculation of heat losses based on heat and mass transfer theory is fairly complicated. Therefore, this is not used by hot water distribution system operators. The paper presents a simpler method based on dimensional analysis [11–14].

## 2. Mathematical modelling

The first phase of modelling considered all the physical parameters on which heat losses depend. In accordance with the rules

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http://dx.doi.org/10.1016/j.measurement.2016.09.033
0263-2241/

concerning the application of dimensional analysis, a dimensional matrix of solutions served as a base to subsequently obtain the dimensionless criteria. These criteria describe the heat losses. Individual values of these criteria are determined from direct measurement on the DN125 network in various operating modes for 12 months. The testing network is above-ground.

In the second phase of the model construction, an interdependence of dimensionless arguments was created, valid for a range of temperatures of the transporting water heating systems (40–70 °C) and the ambient temperature range (from −20 to +30 °C). This temperature range corresponds to winter as well as to summer heat network operation. Based on the drawn-up dependency of dimensionless arguments, it is possible to easily express the heat loss of any heat distribution system. In order to obtain specific values of the intercept constant and the regression coefficient of a given model, it is necessary to follow the procedure recommended and described below.

In the third phase, based on the stated dimensionless arguments, a simplified mathematical model was developed for determining heat losses, particularly suitable for the heating system operators.

Based on the experience of heating network operators and information currently known in literature, from all the relevant variables that influence heat losses in a heat distribution system, those parameters which characterise these losses and which are easily measurable under normal operational conditions were selected.

Heat loss from a pipeline depends upon the quality and thickness of insulation placed on the pipes. The quality of the insulation is contained within the thermal conductivity of the insulation $\lambda_{ins}$. In the mathematical model, the thickness of the insulation is included in the solution via the dimensions which are shown in Fig. 1. In selected relevant variables, the thickness of the insulation itself is not applicable, because it applies that $s_{ins} = (d_3 - d_2)/2$ (Fig. 1). The thermal conductivity of the pipeline material $\lambda_{pi}$ (steel) is irrelevant. This coefficient does not appear in the presented mathematical model. It is used if heat loss is calculated using balance equations based on heat and mass transfer. For all steel pipelines, the value of the mentioned coefficient is considered to be circa 58 W m$^{-1}$ K$^{-1}$. Water with temperature $T_i$ transfers heat to the pipeline and this heat, when using heat transfer theory for calculations, is expressed by the heat transfer coefficient $\alpha_{c,i}$. Heat losses to the ambient environment with temperature $T_e$ are, in the stated method, represented by heat transfer coefficient $\alpha_{c,e}$.

The complete physical equation, expressing the dependence of the relevant variables, can be expressed as

$$\varphi = (P, T_i, T_e, d_1, d_2, d_3, \lambda_{ins}, \upsilon, l) = 0 \quad (1)$$

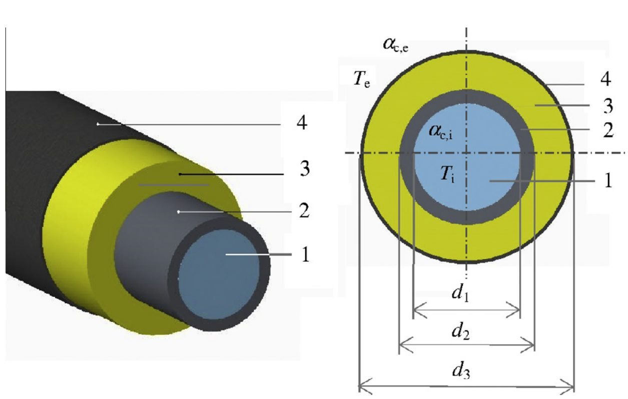


**Fig. 1.** Selected physical parameters for model construction (1 – hot water, 2 – steel pipe, 3 – insulation, 4 – PE-HD outer).

where $P$ – heat power dissipation (W), $T_i$ – transported water temperature of (K), $T_e$ – temperature of the ambient environment (K), $d_1$ – inner diameter of steel pipework (m), $d_2$ – outer diameter of steel pipework (m), $d_3$ – outer diameter of insulation (m), $\lambda_{ins}$ – thermal conductivity of insulation (W m$^{-1}$ K$^{-1}$), $\upsilon$ – water flow velocity (m s$^{-1}$), $l$ – length of pipework (m).

Based on the dimensional diversity of the relevant variables they will create groups, i.e.

$$\pi_i = P^{x_1} \cdot T_i^{x_2} \cdot T_e^{x_3} \cdot d_1^{x_4} \cdot d_2^{x_5} \cdot d_3^{x_6} \cdot \lambda_{ins}^{x_7} \cdot \upsilon^{x_8} \cdot l^{x_9} \quad (2)$$

where from $x_1$ to $x_9$ are exponents. A positive value of the exponent means that the physical quantity will be located in numerator of the dimensionless criteria. In case of negative value of the exponent the physical quantity will be in the denominator.

Dimensional matrix **A** of the given equation with basic units consists of $n = 9$ columns. The number of rows in the matrix corresponds to the number of basic units and equals to $m = 4$. Matrix **A** has the following form

$$\begin{array}{c} \\ \text{kg} \\ \text{m} \\ \text{s} \\ \text{K} \end{array} \begin{array}{c} \begin{array}{ccccccccc} P^{x_1} & T_i^{x_2} & T_e^{x_3} & d_1^{x_4} & d_2^{x_5} & d_3^{x_6} & \lambda_{ins}^{x_7} & \upsilon^{x_6} & l^{x_9} \end{array} \\ \left\| \begin{array}{ccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ -3 & 0 & 0 & 0 & 0 & 0 & -3 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \end{array} \right\| \end{array} \quad (3)$$

If the number of relevant variables $n = 9$ and the level of the matrix $h = 4$, then $i = n - h$, i.e. we can create five (4) dimensionless arguments. Subsequently, a functional dependency will be created. In the given matrix, the dimension of temperature in the selection of the relevant variables occurs twice in total. The dimension of length is found four times. From the above findings, four simplex (similarity criteria) result directly, the form of which is as follows

$$\begin{aligned} \pi_1 &= \frac{T_i}{T_e} \quad (1) \\ \pi_2 &= \frac{d_1}{l} \quad (1) \\ \pi_3 &= \frac{d_2}{l} \quad (1) \\ \pi_4 &= \frac{d_3}{l} \quad (1) \end{aligned} \quad (4)$$

In terms of dimensional analysis, a rectangular matrix (3) can be divided into two parts. The first part of the matrix (**P**) contains the number of columns $h = 8$ and $n = 4$ rows, while the columns of the matrix must be so selected that it possesses a non-zero determinant ($\Delta_{\mathbf{P}} \neq 0$). This corresponds to a vector's distribution of unknown variables $x_i$. Its designation is **R**. For matrix **P** and the vector of unknown variables, **R** applies [5,10]

$$\mathbf{P} \cdot \mathbf{R} = (-1) \cdot \mathbf{Q} \cdot \mathbf{S} \quad (5)$$

where **Q** is the vector of the matrix with a number of columns $h = 1$ and a number of rows $n = 4$. Designation **S** is for the vector of unknown parameters with a number of columns $h = 1$ and a number of rows $n = 1$. Eq. (5), expressed by using Eqs. (2) and (3), can be formulated in the following form (6)

$$\left\| \begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -3 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \end{array} \right\| \cdot \left\| \begin{array}{c} x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{array} \right\| = (-1) \cdot \left\| \begin{array}{c} 1 \\ 2 \\ -3 \\ 0 \end{array} \right\| \cdot \|x_1\| \quad (6)$$