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Robust Switched System Controller Design in the Frequency Domain

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Abstract: A new frequency domain switched controller design procedure is proposed applicable for both SISO and MIMO multi-model systems represented in an affine form. Robust theoretic approach based on the Small Gain Theorem is used to develop a frequency domain condition for closed-loop stability of the multi-model plant in all operation modes as well as stability during switching between individual controllers. Using this condition, a robust switched controller design procedure based the Equivalent Subsystems Method is developed to guarantee robust stability in individual operation modes in addition. The developed design procedure is demonstrated on a case study. The designed robust switched controller has been successfully verified by experiments on a TITO laboratory plant.

Keywords: frequency domain, switched systems, robust stability

1. INTRODUCTION

Switched systems – an important class of hybrid systems are encountered in many industrial applications (control of mechanical systems, automotive industry, aircraft and air traffic control, switching power converters, and many other fields) hence their control has been an important research topic. A switched system consists of several subsystems and a switching rule that specifies the active subsystem in each time instant. Stability of individual subsystems and of arbitrary switching sequence is crucial (Liberzon and Morse, 1999). The majority of existing approaches to switched systems control design are based on time-domain state-space analysis leading to state or static output feedback or high-order dynamic feedback controllers. If the switching signal is not a design variable but belongs to a prescribed admissible set, the problem of feedback stabilization is to find a feedback control law to ensure closed-loop stability under any switching signal. This problem is closely related to the robust control problem of polytopic uncertain linear systems addressable by the Lyapunov approach. Conservatism of these approaches can be reduced using a switched Lyapunov function approach that leads to static output feedback design (Sun and Ge, 2005).

The key approaches to controller design for multi-model systems differentiate in that fulfilment of robust stability conditions guarantees closed-loop stability for any model from the set of systems provided that this model is fixed during operation; however in case of time-varying or switched systems when the model varies during operation, quadratic stability conditions are to be met. According to Anderson (1972), Kunze et al. (2008), Moghaddam and Khaloozadeh (2007) the equivalence between time-domain and frequency-domain conditions for quadratic stability is based upon the link between quadratic stability using Lyapunov theory and Strictly Positive Realness (SPR) properties. In Kunze et al. (2008), a theorem directly related to the equivalence between bounded real lemma and positive real lemma is provided. The resulting design method applicable to guarantee quadratic stability for LPV systems whose scheduling parameter has an affine dependency in the closed-loop expression as well as for switched systems composed of two subsystems each having its specific controller, is restricted to SISO multi-model systems with two subsystems.

Important results on Small Gain Theorem (SGT) can be found e.g. in Anderson (1972), Desoer and Vidyasagar (1975); in Haddad and Bernstein (1991) explicit construction of quadratic Lyapunov functions for the SGT are discussed and asymptotic stability with time-varying memoryless nonlinearity is proved.

The frequency domain methodology proposed in this paper is based on a special affine representation of the switched system and the controller combined with SGT (Veselý and Osuský, 2013a; 2013b), (Kozáková et al., 2014). The developed approach is further extended to the robust decentralized switched system controller design.

The paper is organized as follows. Section 2 presents preliminaries and development of the principle of the proposed SGT-based frequency method; its implementation in the design of robust decentralized switched system controller is presented in Section 3. The design procedure is step-by-step explained on a case study in Section 4; effectiveness of the proposed approach is proved via simulation and experiments on the real plant.

In the sequel, to avoid confusion, the term "subsystem" will denote a constituent part of a complex system whereas a switched system will be specified by several "operation modes (OM)".

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2. PRELIMINARIES AND DEVELOPMENT OF STABILITY CONDITIONS

Consider the standard feedback loop in Fig. 1 consisting of a continuous-time plant $G(s)$ with m inputs and m outputs ($m \geq 1$) and a controller $R(s)$; *w, e, u, d, y* denote vectors of reference, control error, control, disturbance and output, respectively.

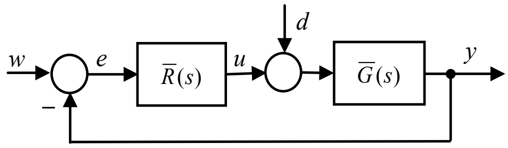

Fig.1. Standard feedback configuration

During operation the plant switches between N different operating modes. Mathematical model of the multi-model plant is given in the affine form (Kozáková et al., 2014):

$$\overline{G}(s) = G_0(s) + \sum_{i=1}^{N} G_i(s) q_i, \; i = 1,2,\ldots,N \tag{1}$$

where $\overline{G}(s), G_0(s), G_i(s)$ are known $m \times m$ transfer function matrices; the scalar $q_i \in \{0, 1\}$ indicates the active operation mode; only one operation mode at a time is supposed to be active, hence $\sum_{i=1}^{N} q_i = 1$.

For example in the k-th operation mode $k \in \{1,2,\ldots,N\}$

$$\begin{aligned} q_k &= 1 \qquad \overline{G}^k(s) = G_0(s) + G_k(s) \\ q_i &= 0 \qquad for \;\; i \neq k \end{aligned}$$

The controller for the multi-model system (1) is supposed to have a similar affine form

$$\overline{R}(s) = R_0(s) + \sum_{i=1}^{N} R_i(s) q_i, \; i = 1,2,\ldots,N \tag{2}$$

where $\overline{R}(s), R_0(s), R_i(s)$ are known $m \times m$ transfer function matrices. The controller has to guarantee closed-loop stability in all operation modes and stable switching between individual controllers. The multi-model system (1) and corresponding switched controller (2) can similarly be written in the matrix-vector form

$$\begin{aligned} \overline{G}(s) &= G_0(s) + Q^T G(s) \\ \overline{R}(s) &= R_0(s) + Q^T R(s) \end{aligned} \tag{3}$$

where

$$R(s) = \begin{bmatrix} R_1(s) \\ \ldots \\ R_N(s) \end{bmatrix}, \; Q = \begin{bmatrix} q_1 I_{m\times m} \\ \ldots \\ q_N I_{m\times m} \end{bmatrix}, \; G(s) = \begin{bmatrix} G_1(s) \\ \ldots \\ G_N(s) \end{bmatrix} \tag{4}$$

$R_i(s)$, $G_i(s)$ are $m \times m$ transfer function matrices and q_i, i = 1,…N are scalars, $R(s)$, $G(s)$ are $Nm \times m$ transfer function matrices. Q is a $Nm \times m$ matrix consisting of stacked $m \times m$ identity matrices each multiplied by the corresponding $q_i \in \{0,1\}$ defining which operation mode is active at the moment. This special structure of Q reflects switching between individual operation modes and corresponding controllers. The plant and controller structures according to (3) are shown in Fig. 2.

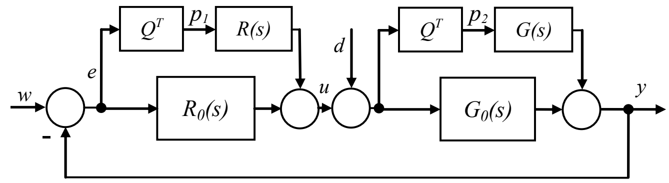

Fig. 2. Feedback loop comprising the switched controller (2) and the multi-model system (1)

In (Haddad and Bernstein, 1991), sufficiency of the small gain theorem for the feedback interconnection of a strictly bounded real transfer function matrix and a norm-bounded memoryless time-varying nonlinearity has been proved. Recall that a transfer function matrix is a real-rational matrix function each of whose entries is strictly proper, and an asymptotically stable transfer function matrix has poles located in the open left half-plane. The space of asymptotically stable transfer function matrices is denoted by $\Re H_\infty$. A transfer function matrix *G(s)* is strictly bounded real if and only if it is asymptotically stable and $\| G(s) \| < 1$. The memoryless time-varying nonlinearity is defined by the set

$$\begin{aligned} &\Phi_{br} \triangleq \{\Phi : R^l \times R^+ \to R^m : \|\Phi(y,t)\|_2 \leq \|y\|_2, \;\; y \in R^l, \\ &where \; t \geq 0, \; \Phi(y,.) \; is \; Lebesque \; measurable \; \forall y \in R^l \; \} \end{aligned} \tag{5}$$

Theorem (*Small Gain Theorem*).

Suppose that $G(s)$ is a strictly bounded real transfer function matrix and $\Phi \in \Phi_{br}$. Then the feedback interconnection of $G(s)$ and Φ is asymptotically stable.

The proof in (Haddad and Bernstein, 1991) is based on state-space representation of the feedback interconnection of *G(s)* and Φ, and explicit construction of the related quadratic Lyapunov function.

To be able to apply the above Theorem, the feedback loop in Fig. 2 has to be recast to the feedback structure in Fig. 3.

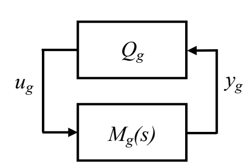

Fig. 3. $M_g - Q_g$ structure

where $y_g = M_g(s) u_g$ (6)

$$y_g = \begin{bmatrix} e \\ u \end{bmatrix}, \quad u_g = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$M_g = \begin{bmatrix} -G_0(I + G_0 R_0)^{-1} R^T & -(I + G_0 R_0)^{-1} G^T \\ (I + R_0 G_0)^{-1} R^T & R_0 (I + G_0 R_0)^{-1} G^T \end{bmatrix} \tag{7}$$

$$M_g \in \Re H_\infty^{2m\times 2Nm}$$