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Feedback Stabilization of Networked Systems over Fading Channels with Resource Allocation ⋆

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Abstract: In this paper, we consider the stabilization problems for networked control systems (NCSs) over parallel independent communication channels, wherein each channel is modeled as a general fading channel, i.e., a combination of multiplicative noise and additive noise. A channel capacity notion is introduced for such general fading channels to represent their capabilities of information transmission. A necessary and sufficient condition for closed-loop stabilization with stationary signal-to-noise ratio (SNR) constraints is established which builds a bridge between the channel capacities and the topological entropy of the open-loop plant. When the condition holds, a channel-controller co-design approach is proposed to accomplish the stabilization. Finally, a numerical example is presented to verify the effectiveness of our results.

Keywords: Networked control systems, feedback stabilization, channel capacity, resource allocation, general fading channel.

1. INTRODUCTION

Network control systems (NCSs) are spatially distributed systems in which the signals between remote sensors, actuators, and controllers are transmitted through a communication network. Such systems have been one of the main research cutting-edges in academia as well as in industry for years, thanks to the fast development of communication technology and the increasing demand of remote control. Generally speaking, the research on NCSs concerns the interplay between three realms: control theory, communication theory, and information theory. The main challenge emerges from the information constraints in the feedback loop due to the imperfect communication network. Fruitful applications of NCSs have been reported in the literature, such as remote surgery, mobile sensor networks, unmanned aerial vehicles and remote diagnostics, etc.

One primary issue of broad interest is the networked stabilization which aims to explore a fundamental limitation on the information constraints in order that the NCS can be stabilized. For a single-input system, such stabilization problems have been extensively studied under different information constraints. See Baillieul et al. (2007); Nair and Evans (2003); Tatikonda and Mitter (2004) for data rate constraint, You and Xie (2010); Silva and Solis (2011) for packet drops, Elia and Mitter (2001); Fu and Xie (2005); Wang et al. (2012) for quantization, Elia (2005) for multiplicative stochastic noise, and Braslavsky et al. (2007) for signal-to-noise ratio constraint, etc. These studies have reached consensus regarding a unified fundamental limitation on the information constraints required for stabilization given in terms of the topological entropy of the open-loop plant, i.e., the logarithm of the absolute product of unstable poles for a discrete-time plant, or the sum of the unstable poles for a continuous-time plant. This builds an elegant bridge between the information requirement and the open-loop dynamics. In fact, as justified from various perspectives in the survey paper Qiu (2010), the topological entropy can be regarded as a measure of the degree of instability of a linear system. Bearing this in mind, it is quite intuitive that the larger topological entropy a system has, the more communication resource is needed for stabilization.

Promising progresses have been made towards the networked stabilization of multi-input systems as well. In particular, the idea of channel/controller co-design was initiated in Qiu et al. (2013) and followed by several other works, for instance, Chen and Qiu (2013); Xiao et al. (2012); Xiao and Xie (2010) to address the multi-input networked stabilization. The essence therein is to employ the twist of channel resource allocation, i.e., assuming that the

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channel capacities can be allocated among different input channels subject to a total capacity constraint. By virtue of this additional design freedom, it has been shown under several different channel models, the minimum total capacity required for multi-input networked stabilization is still given by the topological entropy of the open-loop plant. Most recently, the work in Chen et al. (2014) initiates the use of multiple-input and multiple-output (MIMO) communication in networked control leading to a certain coding/control co-design problem. This matchmaking of MIMO communication and MIMO control brings in new vitality to the investigation of NCSs. A necessary and sufficient condition is obtained for the networked stabilizability given in terms of a majorization condition.

Motivated by these existing results, in this paper, we study the multi-input networked stabilization over general fading channels subject to stationary SNR constraints. Each general fading channel is characterized by the cascade of a multiplicative stochastic noise and an additive Gaussian noise, which is a quite standard setup in communication theory. The work in this paper can be considered as a unification of Qiu et al. (2013) and Xiao et al. (2012) wherein the pure AWGN channels and pure fading channels have been addressed, respectively.

The first issue arising is how to handle the networked stabilization subject to both multiplicative noise and SNR-constrained additive noise. To this end, inspired by the work in Silva and Solis (2011), we attempt to build some sort of equivalence between a multiplicative stochastic noise and a deterministic gain followed by an additive noise subject to an instantaneous SNR constraint. It turns out that as far as mean-square stabilization is concerned, such an equivalence does hold and, thus, our previous experience in networked stabilization over SNR-constrained additive noise channels can be employed. Specifically, we exploit the idea of channel resource allocation which will turn the problem to a channel/controller co-design problem. By investigating the co-design problem, we arrive at a fundamental limitation on the communication channels which is again given by the topological entropy of the open-loop plant.

The remainder of this paper is organized as follows: Section II formulates the problem to be studied in this paper. Section III gives some preliminary knowledge for preparation. The main result is shown in Section IV. A numerical example is worked out for illustration in Section V. Finally, the paper is concluded in Section VI.

NOTATION: The notation used in this paper is mostly standard and will be clarified as we proceed. The notation $\odot$ stands for the Hadamard product, $\mathbb{E}\{\cdot\}$ denotes the mean of a random variable, and $\delta(\cdot)$ represents the Dirac delta function.

2. PROBLEM FORMULATION

Consider a continuous-time system $[A|B]$ described by the state-space model as follows:

$$\dot{x}(t) = Ax(t) + Bu(t),\ x(0) = x_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, and $u(t) \in \mathbb{R}^m$ is the system input. Assume that A is unstable and $[A|B]$ is stabilizable. Also assume that the state $x(t)$ is available for feedback. For conventional control systems, the plant can directly employ the control signal $u(t) = Fx(t)$ with a properly designed gain matrix F to achieve closed-loop stability. However, when it comes to the NCSs wherein the information exchange between the controller and the plant is via a communication network, the feedback control signals from the remote controller may suffer from various distortions or information constraints due to the non-ideal communication channels. This raises new challenges to control analysis and synthesis, even for the basic yet fundamental stabilization problem.

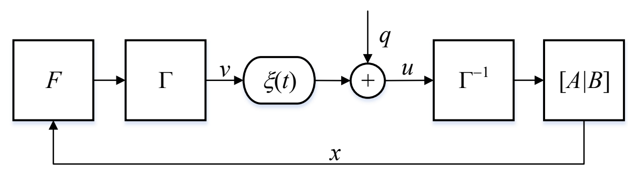

Figure 1. Networked control systems

In this work, we consider the NSCs as in Fig. 1. The communication network is modeled as a set of parallel independent channels such that each control input is transmitted via a dedicated channel. Moreover, a pair of diagonal scaling matrices $\{\Gamma, \Gamma^{-1}\}$ with positive scalar entries are exploited at the transmitting and receiving sides, respectively, where

$$\Gamma = \text{diag}\{\gamma_1, \gamma_2, ..., \gamma_m\}, \gamma_i > 0,\ \text{for } i = 1, ..., m.$$

Such scaling matrices enable the possibility to adjust the transmission power in the different input channels. Similar techniques have been adopted in the literature as well Freudenberg et al. (2007); Qiu et al. (2013); Vargas et al. (2014).

Each channel in the communication network is modeled as a general fading channel whose input-output relation is given by

$$u_i(t) = \xi_i(t)v_i(t) + q_i(t), \tag{2}$$

where the multiplicative noise $\xi_i(t)$ is a white Gaussian process with nonzero mean μ_i and autocovariance $\mathbb{E}[(\xi_i(t) - \mu_i)(\xi_i(t+\tau) - \mu_i)] = \sigma_i^2\delta(\tau)$, while the additive noise $q_i(t)$ is a white Gaussian process with zero mean and autocovariance $\mathbb{E}[q_i(t)q_i(t+\tau)] = p_i^2\delta(\tau)$. The input signal $v_i(t)$ is required to satisfy a stationary power constraint:

$$\mathbb{E}\{v_i^2\} < s_i^2,$$

which in fact places a stationary SNR constraint on the channel, namely, $\mathbb{E}\{v_i^2\}/p_i^2 < s_i^2/p_i^2$. By stacking all the channels together, we can describe the overall communication network as

$$u(t) = \xi(t)v(t) + q(t) \tag{3}$$

where

$$\xi(t) = \text{diag}\{\xi_1(t), \xi_2(t), ..., \xi_m(t)\},$$
$$q(t) = [q_1(t)\ q_2(t)\ \ldots\ q_m(t)]'.$$

Denote

$$M = \text{diag}\{\mu_1, \mu_2, ..., \mu_m\},$$
$$\Sigma^2 = \text{diag}\{\sigma_1^2, \sigma_2^2, ..., \sigma_m^2\},$$
$$Q = \text{diag}\{p_1^2, p_2^2, ..., p_m^2\}.$$