Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-14 (2015) 426–431

Uncertainty Modeling and Robust Analysis of Atmospheric Launchers: Incremental Steps for Industrial Transfer ⋆

Andres Marcos * **Samir Bennani** ** **Christophe Roux** *** **Monica Valli** ***

* *University of Bristol, BS8 1TR, United Kingdom (andres.marcos@bristol.ac.uk).*
** *ESA-ESTEC, Noordwijk, 2201AZ, The Netherlands (e-mail: samir.bennani@esa.int)*
*** *ELV, Rome, 00034, Italy (e-mail: Christophe.Roux@elv.it, Monica.Valli@consultant.elv.it)*

Abstract: Nowadays Linear Fractional Transformations (LFT) and the Structured Singular Value (mu) are very well established concepts for respectively modeling uncertain systems and to perform robust analyses. Despite their ample use in academia and industry during the last 30 years, and the availability of consolidated software toolboxes, their introduction to a new industrial collaborator is not without angst. In this article, the steps followed to transfer this technology to the GNC group at ELV, in charge of the VEGA atmospheric launcher, are presented. The aim of the transfer was to introduce worst-case analysis tools into the VEGA verification and validation process as a complement to the standard LTI gain/phase margin analyses and the nonlinear simulation-based Monte Carlo campaigns. The successful transfer hinged on the reconciliation of the following two facets: (i) the physical behavior of the system with the LFT model capabilities, and (ii) the classical design experience from the VEGA GNC group with the results from the robust mu analyses.

Keywords: LFT modeling, robust analysis, worst-case, launcher

1. INTRODUCTION

A concept widely used in robust control is the structured singular value μ, which analytically evaluates the robustness of uncertain systems Doyle, J. (1982); Doyle, J. et al. (1991); Packard, A. and Doyle, J. (1993); Zhou, K. et al. (1996). A key aspect on the application of μ is the development of a proper LFT model. By proper it is meant a model that captures the critical parametric behavior of the nonlinear system under consideration within a complexity that still enables the application of the μ analysis algorithms.

LFTs and μ have been used in academia and industry during the last 30 years, basically since the appearance of the first version of the toolbox from Balas, G.J. et al. (1998) in July 1993. Its introduction to industry was very quick following a series of hands-on workshops by the developers at a number of companies dealing with complex, uncertain systems, e.g. aerospace. The use of these concepts, methods and tools was consolidated through practice and nowadays is ingrained in those industrial groups that have had the necessity and opportunity, for example for satellites (see references Charbonnel, C. (2010); Pittet, C. and Arzelier, D. (2006)) and the European Automated Transfer Vehicle (ATV), reference Ganet-Schoeller, M. et al. (2009).

Still, despite its wide acceptance and use, it is not without difficulty to try introducing them into the design and analysis process of other industrial groups. Much of this is due to human resources reasons such as: staff rotation, experts' availability and management risk-adverse decisions. But there are also research and development hurdles, which can be summarized in two facets: (i) clear alignment of the physical behavior of the system with the LFT model capabilities, and (ii) reconciliation of classical design experience with the results from the robust μ analyses. In addition, despite well document manuals, detailed tutorials and many publications, there is always the need of a tailored-made benchmark and code scripts that must be transferred to the industrial design group in order for them to really introduce the techniques in their verification and validation (V&V) process. For example, in reference Jang, J.W. et al. (2008), by a renown group of control experts, a simple mass-spring-damper case was used to illustrate the "limitations" of μ in evaluating its potential for the Ares I launcher programme. It was claimed that μ suffered of conservativeness and had to be used with care even for this simple case. But actually, it is easy to show that if a proper LFT model is used then μ correctly identifies a worst-case right on the stability boundary of the (damping, spring) coefficients plane.

Thus, the relevance of this article is precisely in presenting the transfer of these techniques to the GNC group of VEGA, the new European Small Launch Vehicle developed under responsibility of ESA by ELV as the prime contractor –which successfully flew its 4^{th} qualification flight on 11^{th} February 2015. The benchmark selected is a simplified planar launcher motion during atmospheric phase which allows to directly connect the system behavior and results expected by the control experts at ELV with the modeling and robust analysis capabilities of LFT and the structured singular value μ.

⋆ This work was funded by ESA-ESTEC Contract No. 4000111052/14/NL/GLC/al within the project "Analytic Stochastic Time Varying Mu Analysis for the VEGA GNC"

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.09.494

The article layout is based on the three first steps of the process followed to accomplished the transfer:

(1) Provide a high-level theoretical description.
(2) Define a simple, but representative study case.
(3) Follow incremental, step-by-step application.
(4) Release a compact code script of the above.
(5) Collaborate: visit and support.

2. HIGH-LEVEL THEORETICAL PRESENTATION

The importance of a theoretical presentation for industrial transfer is to conceptualize the key ideas underpinning the methods and tools as opposed to provide a detailed and mathematical exact academic exposition.

2.1 Linear Fractional Transformations (LFT)

A LFT is a representation of a system using a feedback interconnection and two matrix operators, $M = [M_{11}\ M_{12};\ M_{21}\ M_{22}]$ and Δ. The matrix M represents the nominal (known) part of the system while Δ contains the parameters ρ_i measuring the unknown. The parameters ρ_i can be real or complex, as well as static, time-varying or nonlinear. There are two possible types of LFTs, lower and upper (see also Fig. 1):

$$F_L(M,\Delta) = M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21} \quad (1)$$

$$F_U(M,\Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12} \quad (2)$$

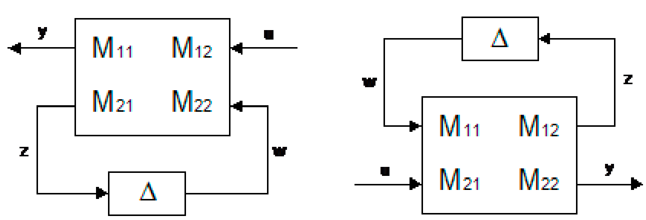

Fig. 1. Lower and upper LFTs

Of course, the LFTs are only well-defined if the inverses exist. The matrix Δ is unrestricted in form (structured or unstructured) but it is important to note that unstructured uncertainty at component level becomes structured at system level. The selection of the variable set $\rho_i \in \Delta$ that captures the behavior of the nonlinear system is a task that is not always obvious a priori. Indeed, this step is key to obtain a LFT that will yield relevant and meaningful results and, despite its apparent simplicity, is where most of the LFT modeling effort and ingenuity is focused. The goal is to achieve the correct trade-off between LFT complexity (number of ρ parameters and total dimension of Δ) versus fidelity with respect to the nonlinear system behaviour.

There are several approaches and toolboxes that facilitate obtaining a proper LFT model (see Lambrechts, P. et al. (1993); Hansson, J. (2003); Magni, J.F. (2004); Marcos, A. and Balas, G.J. (2004); Marcos, A. et al. (2007); Balas, G.J. et al. (2014) and references therein).

2.2 Structured Singular Value, μ

The structured singular value $\mu_\Delta(M)$ of a matrix $M \in C^{n\times n}$ with respect to the uncertain matrix Δ is defined in (1), where $\mu_\Delta(M) = 0$ if there is no Δ satisfying the determinant condition.

$$\mu_\Delta(M) = \frac{1}{\min_\Delta(\sigma(\Delta) : \det(I - \Delta M) = 0)} \quad (3)$$

Note that this definition is given in terms of an $\{M,\Delta\}$ model which is an LFT model where Δ is typically norm-bounded $\|\Delta\|_\infty < 1$ (without loss of generality by scaling of M) for ease of calculation and interpretation. In this manner, if $\mu_\Delta(M) \le 1$ then the result guarantees that the analyzed system, represented by the LFT (for example an uncertain closed-loop system), is robust to the considered uncertainty level. The structured singular value is a robust stability (RS) analysis but can be used also for robust performance (RP), see [4].

Since $\mu_\Delta(M)$ is difficult to calculate exactly, the algorithms implement upper and lower bound calculations Balas, G.J. et al. (1998). The upper bound μ_{UB} provides the maximum size perturbation $\|\Delta_{UB}\|_\infty = 1/\mu_{UB}$ for which RS/RP is guaranteed, while the lower bound μ_{LB} guarantees the minimum size perturbation $\|\Delta_{LB}\|_\infty = 1/\mu_{LB}$ for which RS/RP is guaranteed to be violated. Thus, if the bounds are close in magnitude then the conservativeness in the calculation of μ is small, otherwise nothing can be said on the guaranteed robustness of the system for perturbations within $[1/\mu_{UB}, 1/\mu_{LB}]$.

Note that in the above interpretation, the one used most often, $\mu_\Delta(M)$ becomes a binomial-type of robust analysis (i.e. either the system is robust or not ($\mu_\Delta(M) \le 1$ or > 1)), with an assessment on the conservativeness of the answer given by the range of the bounds, and returning the associated worst-case Δ at these peak values. As it will be shown later this is a simplified view on the analytical power of $\mu_\Delta(M)$ since in reality it is a worst-case frequency-domain analysis allowing to extract robust/worst-case information across frequencies.

3. STUDY CASE FOR LAUNCHER TVC

A launcher thrust vector control (TVC) example is proposed to serve as a simple, yet relevant, study case. The advantage of this case is that it contains some of the main characteristics for atmospheric launchers and facilitates understanding of the results. The (2 rigid states + 1 bending mode of 2^{nd} order) state-space system for this study case is given by:

$$A_{LV} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ A_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_{1B}^2 & -2\zeta_{1B}\omega_{1B} \end{bmatrix};\ B_{LV} = \begin{bmatrix} 0 \\ K_1 \\ 0 \\ -Tn\,TMC_{PVP} \end{bmatrix} \quad (4)$$

$$C_{LV} = \begin{bmatrix} 1 & 0 & -RMC_{INS} & 0 \\ 0 & 1 & 0 & -RMC_{INS} \end{bmatrix};\ D_{LV} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (5)$$

where TMC_{PVP} and RMC_{INS} are the bending mode's translational (at pivot point, PVP) and rotational (at the inertial navigation system, INS) lengths. The general characteristics are:

(1) *Simplified yaw planar rigid motion*. The real-uncertain rigid model is a two states / outputs $[\psi,\ \dot{\psi}]$ containing only the aerodynamic A_6 and controllability K_1 terms and with a single input Tn thrust deflection. (A_6, K_1) are mathematical variables formed by physical parameters such as center of gravity x_{CG}, moment of inertia J_{yy}, dynamic pressure $\bar{q}$, launcher's reference area S_{ref}, yawing coefficient CN_α, center of pressure x_{CP} and pivot reference x_{PVPref}: