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Modelling and Extremum Seeking Control of Gas Lifted Oil Wells

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Abstract This paper addresses the design of a perturbation-based extremum seeking control (ESC) method to maintain the oil production around the optimum point of the well-performance curve in gas lifted wells. The method uses periodic perturbation which is injected into the plant with intention to extract the information about the gradient of the well-performance curve. A simple nonlinear dynamic model is proposed and the essential dynamics of the Eikrem's model are captured, i.e., the transient behavior and the optimal steady state well-performance curve. Based on this simple model, a pre-compensation is developed which allows the application of the ESC scheme without reducing excessively the perturbation frequency. The control performance is evaluated via numerical simulations using an appropriate environment for modelling, simulation and optimization (EMSO) of process dynamic systems.

Keywords: extremum seeking control, oil production, process control, gas lifted wells.

1. INTRODUCTION

In oil production, when the pressure of the reservoir is not enough to maintain the oil flow reaching the wellhead or the well is at the end of its productive life, the use of artificial lift method is required (Thomas, 2001). Hence, artificial lift is a common technique to increase production from mature fields. One of the most used technique is the gas injection (Eikrem et al., 2005) (Skogestad and Storkaas, 2002), which consists of applying compressed gas into the annulus space of the well so that it enters in the bottom of the tubing reducing the average fluid density. As a result, the fluid becomes lighter by decreasing density and allows higher oil flow at the wellhead (Elgsaeter et al., 2010) (Ribeiro, 2012).

The curve representing the relationship between the gas injection flow and the oil flow at the wellhead is named *well-performance curve* (WPC) of gas lifted wells. By increasing excessively the gas flow, the friction increases while the oil production decreases. This results in a WPC with an extremum. The difficulty of estimating the WPC has motivated the search for a robust real-time method that leads to the oil production flow values near the optimum of the WPC (Garcia et al., 2010) (Garcia Irausquin et al., 2008) (Redden et al., 1974) (Aamo et al., 2005).

Optimal lift-gas allocation with constraints was formulated as a mixed-integer nonlinear programming problem in (Camponogara and de Conto, 2002). Optimal allocation of limited resources, such as the lift-gas flow, fluid handling capacities and water-treatment processing capacity was considered in (Camponogara et al., 2009) and (Teixeira, 2013) and a control strategy for the pressure of the gas lift was also proposed. Model predictive control was explored in (Plucenio, 2010) and (Ribeiro, 2012) to assure optimum oil production with quality constrains. Extremum-seeking control (ESC) is an alternative approach for online optimization that deals with uncertain situations when the plant model and/or the cost to optimize are not available to the designer. Using the available signals (plant input and output), the goal is to design a controller that dynamically searches for the optimizing inputs.

In (Ariyur and Krstic, 2003), the method was generalized for a class of dynamic plants stabilizable via state feedback. The general idea was to generate a closed-loop system with sufficiently fast dynamics in order to behave approximately as a static plant. A more general class of nonlinear plants were treated in (Krstić and Wang, 2000) by assuming again that the system (in closed loop via state feedback) can behave approximately as a static one or by assuming that the period of the periodic perturbation is large when compared to the time constant of the system (low excitation frequency). For the class of Hammerstein-Wienner (HW) systems, compensators can be added to the ESC scheme so that the period of the periodic perturbation can be reduced, leading to faster transients to reach the vicinity of the maximizer (Ariyur and Krstic, 2003), (Krstíc, 2000). It must be highlighted that, in all cases (Krstić and Wang, 2000), (Ariyur and Krstic, 2003) and (Krstíc, 2000), the mentioned phase difference is evident. In fact, this ESC method is deeply dependent on a good phase difference detection between input and output for average values of the input below and above the maximizer.

This paper considers the modified version of Eikrem's model by Ribeiro (2012) for gas lifted oil wells. Via

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numerical simulations it is apparent that phase difference is not observed between the input and output of the WPC mapping corresponding to average values of the input below and above the maximizer. A clear phase difference appears only for very low frequencies of the periodic perturbation, which impairs the directly applicability of the ESC method. Seeking to circumvent this problem, in this paper, the modified Eikrem's model is analyzed and a suitable model is proposed. We consider a simple stable first order linear system followed by a nonlinearity containing a product of the plant input and state. This model allows us to capture the main features of the Eikrem' model and clarify the main reason for the difficult to detect the phase difference. Moreover, a simple pre-compensation is proposed in order to approximate the nonlinear system to a HW system, allowing to reduce the period of the periodic perturbation. To the best of our knowledge, perturbation-based extremum seeking control via output feedback has remained unsolved for the class of uncertain nonlinear systems considered here, without reducing the frequency of operation. Hence, as a sub product, this paper also contributes with a preliminar solution to this problem.

Remark 1. (Notation and Terminology) The symbol "s" represents either the Laplace variable or the differential operator "d/dt", according to the context. As in (Ioannou and Sun, 1996), $H(s)u$ denotes the output of a LTI system with transfer function $H(s)$ and input u. Pure convolutions $h(t)*u(t)$, with $h(t)$ being the impulse response from $H(s)$, will be eventually written as $H(s)*u$. Classes of $\mathcal{K}, \mathcal{K}_\infty, \mathcal{L}$ functions are defined according to (Khalil, 2002, p. 144), in particular, a function $\beta : \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{L}$ if it is continuous, strictly decreasing and $\lim_{t\to\infty} \beta(t) = 0$. A function $\alpha : \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$.

2. MODIFIED EIKREM'S MODEL FOR WELL PRODUCTION

According to (Eikrem et al., 2002), the model to describe the well production (3 phases – water, oil and gas) is given in four parts: (i) mass balance model of the phases, (ii) the densities models, (iii) the pressures models and (iv) the flows models. It is assumed that the oil and water form one single phase (inside the well column) and only slow changes occurs in the quantity of gas in the mixture. The mass balance of the production process of a single well operating via gas lift can be described by:

$$\dot{x}_1 = u - w_{iv}\,, \tag{1}$$
$$\dot{x}_2 = w_{iv} - w_{pg}\,, \tag{2}$$
$$\dot{x}_3 = w_{ro} - w_{po}\,, \tag{3}$$
$$y = w_{po}\,, \tag{4}$$

where, x_1 is the mass of gas in the annulus, x_2 is the mass of gas in the tubing, x_3 is the mass of oil production in the column above the injection point, $u = w_{gc}$ is the flow gas injection in the annulus (control input), w_{iv} is the flow gas from the annulus to the tubing, w_{pg} is the gas flow though the production valve (choke), w_{ro} is the oil flow from the reservoir into the tubing, and $y = w_{po}$ is the oil flow though the production choke (plant output).

The oil density in the reservoir is given by $\rho_0 = \frac{1}{v_0}$, where v_0 is the specific volume of the oil in the reservoir (the oil is considered incompressible). The densities ρ_{ai} (gas density in the annulus at the injection point), ρ_m (density of the oil and gas mixture at the wellhead) are described by:

$$\rho_{ai} = \frac{M}{RT_a}\left(\frac{RT_a}{V_a M} + \frac{gL_a}{V_a}\right)x_1, \tag{5}$$
$$\rho_m = \frac{x_2 + x_3 - \rho_o L_r A_r}{L_w A_w}, \tag{6}$$

respectively, where R is the universal constant of the ideal gases, T_a is the temperature in the annulus, V_a the volume of the annulus space, M is the molar mass of the gas, L_a the length of the annulus, g is the acceleration of gravity, A_w is the cross section area (assumed circular) of the column above the injection point, L_w is the length of the column above the injection point, A_r is the cross section area (assumed circular) of the column below the injection point and L_r is the length of the column below the injection point.

According to the modifications in the Eikrem's model proposed by Ribeiro (2012), the pressure p_{ai} (the annulus pressure at the injection point of the column), p_{wh} (well-head pressure), p_{wi} (column pressure at the injection point of the column) and p_{wb} (downhole pressure) are given by:

$$p_{ai} = \left(\frac{RT_a}{V_a M} + \frac{gL_a}{V_a}\right)x_1\,, \tag{7}$$
$$p_{wh} = \frac{RT_w}{M}\frac{x_2}{L_w A_w + L_r A_r - v_o x_3}\,, \tag{8}$$
$$p_{wi} = p_{wh} + \frac{g}{A_w}(x_2 + x_3 - \rho_o L_r A_r) + \rho_0 g h_f(L_w)\,, \tag{9}$$
$$p_{wb} = p_{wi} + \rho_o g(L_r + h_f(L_r))\,, \tag{10}$$

respectively, where T_w is the temperature in the column, $h_f(\cdot)$ is the head loss described in Ribeiro (2012). The flows w_{iv}, w_{pg}, w_{po} and w_{ro} are described by:

$$w_{iv} = C_{iv}\sqrt{\rho_{ai} max\{0, p_{ai} - p_{wi}\}}, \tag{11}$$
$$w_{pg} = \frac{x_2}{x_2 + x_3}w_{pc},\ \ w_{po} = \frac{x_3}{x_2 + x_3}w_{pc}\,, \tag{12}$$
$$w_{ro} = C_r\sqrt{\rho_0(p_r - p_{wb})}, \tag{13}$$

where,

$$w_{pc} = C_{pc}\sqrt{\rho_m max\{0, p_{wh} - p_s\}}, \tag{14}$$

C_{iv}, C_{pc} and C_r are positive constants, p_s is pressure in manifold downstream of the well where it is assumed that there is a control to maintain this pressure at a constant value, and p_r is the reservoir pressure far from the well, which is also considered constant.

Note that, from (12), one has that $w_{pg} = (x_2/x_3)w_{po}$. Therefore, the system (1)–(4) can be rewritten as follows:

$$\begin{aligned} \dot{x}_1 &= u - \varphi_1(x_1, x_2, x_3), \\ \dot{x}_2 &= \varphi_1(x_1, x_2, x_3) - \frac{x_2}{x_3}\varphi_3(x_2, x_3), \\ \dot{x}_3 &= \varphi_2(x_2, x_3) - \varphi_3(x_2, x_3), \\ y &= \varphi_3(x_2, x_3), \end{aligned} \tag{15}$$

where $\varphi_1 = w_{iv}$ is obtained from (11), (5), (7), (9) and (8), $\varphi_2 = w_{ro}$ is obtained from (13), (10), (9) and (8) and $\varphi_3 = w_{po}$ is obtained from (12), (14), (6) and (8).