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Production Optimization under Uncertainty with Constraint Handling

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Abstract: To maximize the daily production from an oil and gas field, mathematical optimization may be used to find the optimal operating point. When optimizing, a model of the system is used to predict the outcome for different operating points. The model is, however, subject to uncertainty, e.g., the gas oil ratio estimates may be imprecise. The uncertainty is often ignored, and what is known as the expected value problem is solved. Because of inherent uncertainties, there is a great chance that constraints will be violated when implemented. In this paper, we formulate the production optimization problem as a stochastic programming problem, and use Conditional Value at Risk to handle the constraints. This allows us to control the conservativeness of the solution in an efficient manner.

Keywords: Production optimization, uncertainty, stochastic programming

1. INTRODUCTION

During exploitation of hydrocarbon resources, a wide range of decisions are made on how to produce the field. They range from choosing equipment to deciding on choke positions and gas lift rates for the different wells. These decisions will affect the production and profitability of the field, and there is a growing interest in using optimization tools for decision support to increase the profitability. The term Real-Time Optimization (RTO) is used in the oil and gas industry about processes which include some sort of mathematical optimization to maximize profit. An overview of RTO within oil and gas production systems can be found in Bieker et al. (2006).

Since the production system and reservoir is a complex system, it is difficult to optimize everything simultaneously. However, the process contains parts with highly different time constants; in particular the reservoir evolves slowly compared to the dynamics of valves and pipelines. This allows for a hierarchical treatment when controlling the process. In Foss and Jensen (2011), this hierarchy is divided into the four layers Asset Management, Reservoir Management, Production Optimization, and Control and Automation. In this work, we concentrate on production optimization, however, it is closely linked to the other layers of the hierarchy, and especially reservoir management.

In most of the reported industry implementations, production optimization is done without considering the uncertainty of model parameters. Unfortunately, the quantities used in such an optimization problem are seldom known precisely. For instance, the gas oil ratio (GOR) and water cut (WC) of wells can be quite uncertain, due to sparse well tests, changing operating conditions and measurement errors. Although they are known to be uncertain, the optimization problem would typically be solved using the most likely GOR and WC, which could be the values obtained from the last well test. This leads to what is known as the expected value solution. This is an intuitive approach, however, it neglects the inherent uncertainty of the problem. It was pointed out in Bieker et al. (2007a); "The handling of model uncertainty is a key challenge for the success of RTO".

When introducing uncertainty in the optimization problem, the objective function can be expressed as a function of the decision variables and the unknown parameters. We write $J(x, \omega)$, where x is the vector of decision variables and ω is the vector of unknown parameters. ω is stochastic, hence the objective function is also stochastic. Thus, for a given decision x, we can not determine the exact outcome, because it is also dependent on the unknown parameters ω. For an unconstrained problem, the expected value solution can be obtained by solving

$$\min_x J(x, \mathbb{E}[\omega]) \tag{1}$$

where $E[\omega]$ denotes the expected value of ω. We denote this as the deterministic problem. However, this approach basically ignores the uncertainty in the parameters. What we are really interested in, is solving the stochastic problem, which can be expressed as

$$\min_x \mathbb{E}[J(x, \omega)] \tag{2}$$

when using a risk neutral preference. Note that in general, $\mathbb{E}[J(x, \omega)] \neq J(x, \mathbb{E}[\omega])$.

The stochastic problem is significantly harder to solve than the deterministic problem, since evaluating the objective function involves multidimensional integration. As a con-

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sequence of this, approximate algorithms are often used, for instance by sampling ω, we can approximate

$$\mathbb{E}[J(x,\omega)] \approx \frac{1}{N}\sum_{i=1}^{N} J(x,\omega_i) \quad (3)$$

where ω_i represents different realizations, all with the same probability. This is known as the sample average approximation (SAA).

One of the challenges when handling the uncertainty, is the need to describe, or actually model, the uncertainty. It is no longer enough to provide a reasonable estimate of the parameters, it must also be specified "how certain" they are. One way of doing this is to provide the probability distribution function of the parameters, however, they are seldom known precisely. There exists techniques for estimating the uncertainty. We will assume that such a technique is available since the focus of this work is the optimization problem. In Elgsæter et al. (2008), bootstrapping is used for obtaining parameter and uncertainty estimates.

We first give an overview of previous work in Section 2, before focussing on stochastic programming in Section 3. We then introduce a case study with results in Section 4 and 5, before a discussion and conclusion in Section 6.

2. PREVIOUS WORK

As mentioned, most of the earlier work on RTO ignore uncertainty, and thus solve what is known as the expected value problem. There are, however, a few publications treating the uncertainty explicitly, particularly in the reservoir management domain.

2.1 Work on reservoir management under uncertainty

In Aitokhuehi and Durlofsky (2005), a small number of geological realizations is used to optimize the average recovery factor in closed loop reservoir optimization. A risk term is also used in the objective.

van Essen et al. (2009) optimize the expected NPV by controlling the water injection. An ensemble of 100 realizations is used for the test case of 8 injection and 4 production wells, showing that this approach significantly improves the average NPV compared to approaches using only a single reservoir model.

Chen et al. (2009) combine an ensemble-based optimization scheme with the ensemble Kalman filter for closed loop reservoir optimization. The method uses the ensemble for estimating the gradient, eliminating the need for adjoints and thus any reservoir simulator can be used. An example where the expected NPV is optimized for a water flooding scenario is given.

In Alhuthali et al. (2010), waterflooding is optimized by minimizing the expected deviation from desired arrival time at the producers over a set of realizations. An approach minimizing the maximum deviation is also used. Chen et al. (2011) use a robust scheme to combine short and long term optimization. The long term solution is obtained by optimizing the expected NPV for an ensemble of reservoir realizations, and is used as a constraint in the short term problem. The short term problem involves a more heavily discounted expected NPV over a short time horizon, with a constraint limiting the decrease in the long term expected NPV. Operating constraints are included in a robust fashion, so all constraints must be satisfied for all realizations.

Wang et al. (2012) optimize well placement under uncertainty, using retrospective optimization to limit the number of realizations needed. The number of realizations are gradually increased in the algorithm. Li et al. (2012) optimize both the placement and operation of wells using simultaneous perturbation stochastic approximation to reduce the cost associated with gradient evaluation.

Capolei et al. (2013) compare the solution from a stochastic formulation to the certainty equivalence solution, when the model ensembles are updated based on measurements from a true model. They conclude that when updating the model ensemble, the certainty equivalence approach is superior to the stochastic solution. The comparison is, however, only done for 1 or 2 realizations, and not all of the realizations. A different choice of the true model could thus result in another conclusion.

Dilib and Jackson (2013) use an approach where the parameters of a closed loop controller is optimized to maximize the NPV of the nominal case, and their results suggest this can reduce the effect of uncertainty. The test case is, however, quite simple, and their conclusion can not be generalized.

2.2 Work on short term production optimization under uncertainty

Although there exists numerous publications for reservoir management under uncertainty, there are only a few published papers on short term production optimization under uncertainty. In Elgsæter et al. (2010), a structured approach for changing the setpoint when there is uncertainty is proposed. The uncertainty is, however, not considered in the optimization itself, only to assess the solution from the optimization. To our knowledge, the only publication where the uncertainty is explicitly handled in the optimization problem is by Bieker et al. (2007b). They propose to formulate the optimization problem as a priority list between the wells. This list represents an operational strategy the operator should follow, and whenever there is spare capacity or production must be decreased, he should follow the list. Assuming that all wells are closed, the highest priority well should be opened until it is fully open, or a constraint is hit. If there is still more capacity left, the operator should open the second highest priority well and so on.

3. STOCHASTIC PROGRAMMING

When optimizing systems containing uncertainty, it is often natural to use the expected value of the objective function by averaging over the different realizations. Often the system will also be subject to some limitations, which are modeled as constraints in the optimization problem. For the deterministic problem, they are usually expressed as $c(x) \leq 0$, but if the constraints are uncertain, they will also depend on the unknown parameters ω. There are