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Profit and Risk Measures in Oil Production Optimization ⋆

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Abstract: In oil production optimization, we usually aim to maximize a deterministic scalar performance index such as the profit over the expected reservoir lifespan. However, when uncertainty in the parameters is considered, the profit results in a random variable that can assume a range of values depending on the value of the uncertain parameters. In this case, a problem reformulation is needed to properly define the optimization problem. In this paper we describe the concept of risk and we explore how to handle the risk by using appropriate risk measures. We provide a review on various risk measures reporting pro and cons for each of them. Finally, among the presented risk measures, we identify two of them as appropriate risk measures when minimizing the risk.

Keywords: Production Optimization, Oil production, Risk measures, Stochastic problems

1. INTRODUCTION

In oil production optimization, we are in general interested in maximizing an economic measure, like the profit or the net present value (NPV), over the expected reservoir life time. When uncertainty is taken into account, the profit is not a single quantity but has a probability distribution, i.e. the profit is described by a random variable ψ. An optimization problem involving ψ in terms of a control input u, must express ψ as a scalar quantity. Traditionally, this single quantity is the expected profit (Van Essen et al., 2009; Capolei et al., 2013). By using only the expected profit, however, we are not able to include others important indicators, that shape the profit distribution ψ, such as the profit deviation and the risk preference. The role of a measure of deviation is to quantify the variability of a random variable ψ and the uncertainty in ψ is often measured by the standard deviation of ψ, e.g. in classical portfolio theory (Markowitz, 1959), the standard deviation $\sigma(\psi)$ is used to quantify uncertainty in returns of financial portfolios. In the oil community, Bailey et al. (2005); Alhuthali et al. (2010); Yeten et al. (2003) propose to reduce the uncertainty in profit by including the standard deviation in the cost function. In many decision problems dealing with safety and reliability, risk is often interpreted as the probability of a dreadful event or disaster (Ditlevsen and Madsen, 1996; Rockafellar and Royset, 2010), and minimizing the probability of a highly undesirable event is known as the safety-first principle (Roy, 1952). In this paper we identify the risk as a measure of the risk of loss. When speaking of such a measure applied to the random profit, ψ, we have in mind that higher outcomes of ψ are welcome while lower outcomes are disliked. To reduce the risk of loss then, we seek to lower the probability of the low profits. Certainly, deviation and risk are related concepts and often these terms are used interchangeably, e.g. in finance, we can interpret the profit volatility, measured by the standard deviation, as risk. Following this idea, Capolei et al. (2015) introduce the mean-variance criterion for production optimization and suggest to use the Sharpe ratio as a systematic procedure to optimally trade-off risk and return. They interpret the standard deviation as a measure of risk. However, the mean-variance approach is more suited to reduce the profit uncertainty than to reduce the risk of loss. Fig. 1 illustrates two drawbacks of the mean-variance framework when used to measure risk preferences. First of all, the mean variance approach is insensitive to the profit shape distribution. Fig.1a is a sketch representing different profit distributions having the same values for mean and the variance. In the mean-variance framework these distributions yield the same risk preference. In Fig.1b instead, the distributions in blue have a lower standard deviation, σ, than the distribution in black. If we use the standard deviation as a risk measure, the blue distributions have a lower risk than the black profit distribution, no matter what their expected values are. Furthermore, the standard deviation as a measure of risk is symmetric, which means that it penalizes higher profits and lower profits symmetrically. This last shortcoming have been recognised by Markowitz (1959) who proposed to use the semideviation instead. However, even by using the semideviation, we still do not have common properties that make sense both for a risk

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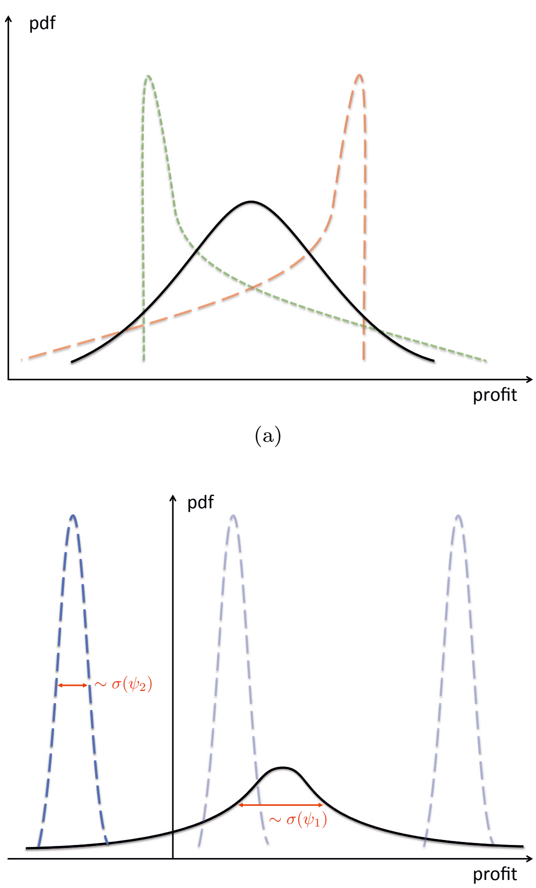
measure like risk aversion and monotonicity, see Section 3.

In this paper we follow an axiomatic approach to define risk, i.e. we first define the principles that an appropriate risk measure should have, then we select risk measures that satisfies such principles. The risk axioms that we use are the principles that define coherent averse risk measures as introduced and defined in Artzner et al. (1999); Rockafellar (2007); Krokhmal et al. (2011); Zabarankin and Uryasev (2014). At our knowledge, this is the first time that such an axiomatic approach is used in oil production optimization.

The paper is organized as follow. Section 2 formulates the oil production optimization problem under uncertainty as a risk minimization problem. Section 3 describes the basic properties that we require from an appropriate risk measure, while a number of risk measures are discussed in Section 4. Conclusions are presented in Section 5.



(a)

(b)

Fig. 1. The mean-variance framework is indifferent to shape distributions. Fig.1a is a sketch representing different profit distributions having the same mean and variance. In the mean-variance framework, these distributions yield the same risk preference. In Fig.1b, the distributions in blue have a lower standard deviation σ than the distribution in black. If we use the standard deviation as a risk measure, this means that the blue distributions have a lower risk than the black profit distribution, no matter what their expected values are.

2. PROBLEM FORMULATION

In oil production optimization, the profit can be visualized as a function

$$\psi = \psi(u, \theta) \tag{1}$$

of a decision vector $u \in \mathcal{U} \subset \mathcal{R}^{n_u}$ representing the control vector, with $\mathcal{U}$ expressing linear decision constraints, and a vector $\theta \subset \mathcal{R}^m$ representing the values of a number of parameters variables such as the permeability field, porosity, economic parameters, etc. The function ψ usually represents the NPV or some other performance index, and its computation typically requires the use of a reservoir simulator to solve the reservoir flow equations. If there is no uncertainty in the parameters values θ, we can maximize ψ by solving the following deterministic optimal control problem (Brouwer and Jansen, 2004; Sarma et al., 2005; Nævdal et al., 2006; Foss and Jensen, 2011; Capolei et al., 2013)

$$\max_{u \in \mathcal{U}} \psi(u, \theta) \tag{2}$$

However, in oil problems there is a high uncertainty due for example to the noisy and sparse nature of seismic data, core samples, borehole logs, and future oil prices and plant costs. Mathematically, we may represent model uncertainty by making the parameter vector θ a random variable that has some probability distribution and that belongs to some uncertainty space Θ. Consequently, the profit ψ is a random variable. Due to the complexity of real oil reservoirs and the accompanying measurement problem, we don't know the probability distribution of θ, thus, we have only incomplete informations about the uncertainty space Θ. For these reasons, the traditional way of modeling the uncertainty in oil production problems is to consider a finite set of possible scenarios for the parameters (Krokhmal et al., 2011; Van Essen et al., 2009; Capolei et al., 2013, 2015). This means that we substitute Θ with the discretized space $\Theta_d := \{\theta_1, \theta_2, \ldots, \theta_{n_d}\}$. As a consequence, a control input u, will correspond to a finite set of possible profit outcomes $\psi(u, \theta_1), \ldots, \psi(u, \theta_{n_d})$, with probabilities $p_1, \ldots, p_{n_d}$, respectively, where $p_i = Prob[\theta = \theta_i] \in [0, 1]$ and $\sum_{i=1}^{n_d} p_i = 1$. Usually the possible realizations θ_i are considered equiprobable, i.e. $p_i = 1/n_d$. It should be noted that defining the uncertainty set Θ_d is a highly interdisciplinary exercise. Furthermore, uncertainty will be updated as subsurface properties further reveal themselves from measurement surveys and production data in addition to new forecasts on oil price and costs.

When uncertainty is taken into account, the following stochastic optimization problem can be written

$$\max_{u \in \mathcal{U},\, \theta \in \Theta_d} \psi(u, \theta) \tag{3}$$

However, this formulation is not well defined. The problem is that the decision vector u must be chosen before the outcome of the distribution of θ and consequently the value of ψ, can be observed. To obtain a well defined problem, we can substitute the random variable ψ by a functional $\mathcal{R} : \psi \to \mathbb{R}$ that yields a scalar measure of ψ, and depends both on u and θ. In this way we can reformulate problem (3) as

$$\min_{u \in \mathcal{U}} \mathcal{R}(\psi(u, \theta \in \Theta_d)) \tag{4}$$

Note that we have switched to a minimization problem because we will interpret $\mathcal{R}$ as a risk measure to minimize.