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Robust output-feedback control to eliminate stick-slip oscillations in drill-string systems

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Abstract: The aim of this paper is to design a robust output-feedback controller to eliminate torsional stick-slip vibrations. A multi-modal model of the torsional dynamics with a nonlinear bit-rock interaction model is used. The controller design is based on skewed-μ DK-iteration and the stability of the closed-loop nonlinear system is analyzed. The proposed controller design strategy offers significant advantages compared to existing strategies. First, it requires only surface measurements, second, it can effectively deal with multiple torsional flexibility modes, third, it provides robustness with respect to uncertainties in the bit-rock interaction and finally, control performance specifications can be taken into account. Simulation results confirm that stick-slip vibrations are indeed eliminated using the designed controller.

Keywords: Stick-slip oscillations, robust control, μ-synthesis, output-feedback, drilling systems.

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1. INTRODUCTION

Efficiency, reliability, and safety are important aspects in the drilling of deep wells for the exploration and production of oil, gas, mineral resources, and geo-thermal energy. Drill-strings of several kilometers in length are used to transmit the axial force and torque necessary to drill the rock formations. These drill-string systems are known to exhibit different types of self-excited vibrations, which decrease the drilling efficiency, accelerate bit wear, and may cause drill-string failure due to fatigue.

Modelling of the torsional dynamics of the drill-sting is an important step towards the control of torsional vibrations. Most controller designs presented in literature rely on one- or two degree-of-freedom (DOF) models for the torsional dynamics only, see e.g. Jansen and Van den Steen (1995); Tucker and Wang (1999); Serrarens et al. (1998); De Bruin et al. (2009). The resisting torque-on-bit (TOB) is typically modelled as a frictional contact with a velocity weakening effect. Although experiments using single cutters to identify the bit-rock interaction law (Detournay and Defourny, 1992) do not reveal such a velocity weakening effect, analysis of models that take the coupled axial and torsional dynamics into account show that such coupling effectively leads to a velocity weakening effect in the TOB (Richard et al., 2007). This motivates a modelling-for-control approach involving the torsional dynamics only and a velocity weakening bit-rock interaction law. In contrast to other studies, however, we use a multi-modal model of the torsional dynamics as field observations have revealed that multiple torsional resonance modes play a role in the onset of stick-slip oscillations.

Controllers for drilling systems aim at drill-string rotation at a constant velocity and the mitigation of stick-slip vibrations. Moreover, the following control specifications are important. First, only surface measurements can be used for feedback. Second, the controller should be able to cope with dynamics related to multiple torsional flexibility modes. Third, robustness with respect to uncertainty in the bit-rock interaction has to be guaranteed and, fourth, control performance specifications, related to e.g. measurement noise sensitivity and actuator constraints, need to be taken into account in the control design.

A well-known control method, which aims at damping the first torsional mode, is the *Soft Torque Rotary system* (Halsey et al., 1988). The same objective is set in Jansen and Van den Steen (1995), which uses a PI-controller based on the top drive velocity. Other control methods including, torsional rectification (Tucker and Wang, 1999), observer-based output-feedback (De Bruin et al., 2009; Doris, 2013), weight-on-bit control (Canudas-de Wit et al., 2008) and robust control (Serrarens et al., 1998; Karkoub et al., 2010) have been developed and are documented in literature.

Although important steps have been made, an approach that satisfies all mentioned requirements has not yet been developed. A robust control approach, as proposed in the latter two works, is particularly suitable for this problem since both robustness with respect to uncertainty of the

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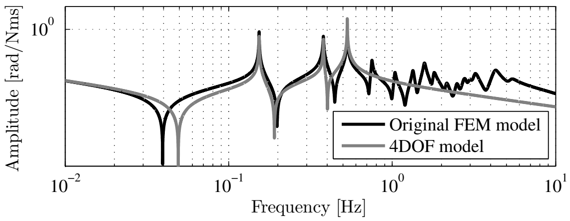

Fig. 1. Frequency response function of the FEM model and 4-DOF model from bit torque v to bit velocity ω_1, i.e. bit mobility.

system dynamics and control performance specifications can be taken into account in the control design. In Serrarens et al. (1998), an $\mathcal{H}_\infty$ controller synthesis method is applied to a 2-DOF drill-string model, using the twist in the drill-string is used as measurement, i.e. knowledge about the angular position of the bit is assumed to be known. Karkoub et al. (2010) uses the μ-synthesis technique through DK-iteration procedure to obtain less conservative bounds on the uncertainty to obtain robustness with respect to the nonlinear bit-rock interaction. The used model is a similar 2-DOF model and also in this case down-hole measurements (to asses the twist of the drill-string) are used. Moreover, the employed 2-DOF models only take the first flexibility mode into account.

The main contribution of this paper is the design of a robust output-feedback controller methodology to eliminate stick-slip vibrations, with the following advantages over existing controllers: 1) usage of surface measurements only, 2) application of the controller to multi-modal drill-string models while guaranteeing local stability of the desired set-point, 3) optimization of the robustness with respect to uncertainty in the bit-rock interaction and, 4) integration of control performance specifications in the design approach.

2. DRILL-STRING MODEL

A lumped-parameter model that represents a drilling system is proposed as a basis for controller design. The proposed model is based on a finite element method (FEM) model representation of a realistic drilling system (representing a discretization of a distributed parameter (PDE) model of the drill-string dynamics), see Vromen et al. (2014) for more details on the FEM model. The bit mobility (see Fig. 1) gives an indication of the important resonance modes in the onset of stick-slip vibrations, it is clearly visible that the first three resonance modes are dominant. Therefore, a 4-DOF model is developed, which incorporates 4 rotating inertias, connected to each other with springs and dampers to model the torsional flexibility and damping characteristics of a drill-string (see Fig. 2). The lower disc represents the drill bit in practice, the upper disc the top drive inertia, and the other degrees of freedom characterize additional flexibility modes. The inclusion of these extra modes in the model is a key improvement with respect to existing models used for controller design.

The driving input of the system is the motor torque T_m. The available measurements are the top drive velocity ω_{td} and the pipe torque T_{pipe}, which is defined as the torque in the drill-string right below the top drive. The drill-string-borehole interaction torques $\phi_i(\omega_i)$, $i = 2, 3, 4$, are modelled as set-valued Coulomb friction laws ($\phi(q_2) := [\phi_2(\omega_2)\ \phi_3(\omega_3)\ \phi_4(\omega_4)]^\top$, with $q_2 := [\omega_2\ \omega_3\ \omega_4]^\top$) and the interaction torque $\varphi(\omega_1)$ at the bit-rock interface is defined by a set-valued Coulomb friction law with Stribeck effect, see Fig. 2. The resulting equations of motion are written in first-order state-space form:

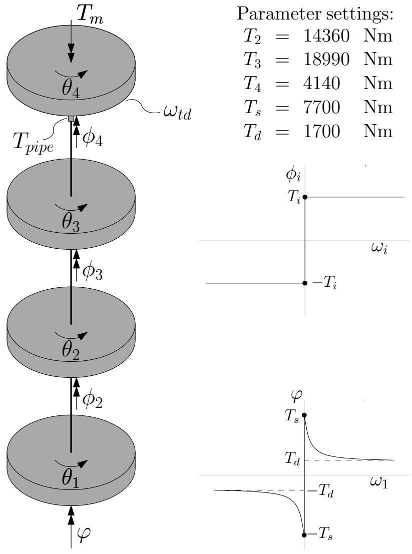

Fig. 2. 4-DOF model of the drill-string.

$$\begin{aligned}
\dot{x} &= Ax + Gv + G_2 v_2 + Bu_t \\
q &= Hx \\
q_2 &= H_2 x \\
y &= Cx \\
v &\in -\varphi(q) \\
v_2 &\in -\phi(q_2).
\end{aligned} \tag{1}$$

Herein, $x = [\theta_1 - \theta_2,\ \omega_1,\ \omega_2,\ \theta_2 - \theta_3,\ \omega_3,\ \theta_3 - \theta_4,\ \omega_4]^\top \in \mathbb{R}^7$ is the state, where θ_i, $i = 1, 2, 3, 4$, describes the rotational displacement of the inertias, $\omega_i := \dot{\theta}_i$, and the bit velocity is defined as $q := \omega_1$. Moreover, the bit-rock interaction torque is given by $v \in \mathbb{R}$ and the drill-string-borehole interaction torques are given by $v_2 \in \mathbb{R}^3$. In addition, $u_t := T_m \in \mathbb{R}$ is the control input and, $y := [\omega_{td}\ T_{pipe}]^\top \in \mathbb{R}^2$ is the measured output.

3. CONTROL PROBLEM FORMULATION

The desired operation of the drill-string system is a constant angular velocity ω_{eq} for all four inertias. So, the objective is to regulate this set-point of the nonlinear drill-string system by means of an output-feedback controller. The available measurements for the controller are the top drive angular velocity ω_{td} and the pipe torque T_{pipe}. The system can be controlled by the top drive torque T_m. As briefly mentioned in the introduction, the controller should

(1) locally stabilize the desired velocity of the drill-string, therewith eliminating torsional (stick-slip) vibrations;
(2) ensure robustness with respect to uncertainty in the nonlinear bit-rock interaction φ;
(3) guarantee the satisfaction of closed-loop performance specifications, in particular on measurement noise sensitivity, i.e., limitation of the amplification of measurement noise, and limitation of the control action such that top drive limitations can be satisfied;
(4) guarantee robust stability and performance in the presence of multiple flexibility modes dominating the torsional dynamics.