Measurement 90 (2016) 224–232

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Energy density for signals maximizing the integral-square error

Krzysztof Tomczyk *, Edward Layer

Cracow University of Technology, Faculty of Electrical and Computer Engineering, Poland

ARTICLE INFO

Article history:
Received 29 December 2015
Received in revised form 15 March 2016
Accepted 25 April 2016
Available online 28 April 2016

Keywords:
Wavelet transform
Energy density
Scalogram
Signal constrains
Signals maximizing integral-square error

ABSTRACT

The paper presents an application of the continuous wavelet transform (CWT) to analyse the energy density of signals which maximize the integral-square error (ISE) at the output of two different accelerometers. These accelerometers were chosen as being examples of a large class of measuring instrument intended for the measurement of nondetermined input signals. Input signals constrained in magnitude only and simultaneously in magnitude and rate of change are considered in this paper.

Scalogram analysis which provides a graphical representation of the signal energy density over the time-scale plane is discussed in detail in the second section of this paper. In the third, the results for the two accelerometers modelling are presented, while the fourth section presents methods for determining signals maximizing the ISE by using the genetic algorithm (GA).

The final sections is devoted to a discussion of the results and analysis of the energy density based on the scalogram and corresponding conclusions with respect to properly determined signals maximizing ISE.

For modelling the sensors MathCad15 was applied, while the maximizing signals and CWT analysis were executed using MATLAB2011.

The methods presented in this paper constitute a novel approach for the estimation of the correctness of the signals maximizing the ISE by means of energy density analysis.

1. Introduction

Sensors play a very important role in the theory and practice of measurement in various fields such as biology, medicine, physics, mechanics and many others. Depending on their field of use, specific requirements are imposed upon sensors with regard to accuracy. In the case of measuring instruments intended for static measurement the class index resulting from the value of maximum static error indicates their accuracy. For a long time this index has been determined by the process of calibration, controlled by legal regulations which define the hierarchy of standards and calibration procedures.

In the case of sensors intended for measurement of dynamic signals, calibration process similar to those available for static measuring instruments have not yet been elaborated till now. The methods used here were usually based on the testing of sensor responses on Dirac impulse, unit step, or a typical input testing signal e.g. ramp function or a square wave in time domain, or in the frequency domain on frequency characteristics of the sensor under test with a rather free interpretation of the obtained results [1–4]. The problem of signals selection based on the determination of dynamic errors appears only in 2008 in the document of the JCGM 100:2008 (Committee for Guides in Metrology), where in p.04 we find the following recommendation: "... *result of a measurement should be universal: the method should be acceptable to all kinds of measurements and to all types of input data used in measurements*" [5]. As it is impossible to analyse the full range of all possible dynamic signals, we will determine the one signal that we will then consider as being representative of all the signals that we are interested in. The signal we chose will be the one generating an error of maximum value. This signal can be taken as including or representing all other signals and a signal of any shape, occurring at the input of the sensor, will always generate an error of less than or at most equal to this maximum value [6–8].

A signal maximizing the dynamic error should be matched to the dynamic characteristics of the sensor under test, as it can transmit signals only with a finite rate of change [6–9]. It follows from the fact that the dynamics of all kind of low-pass sensors are limited and results from frequency characteristics as well as form their band pass range. Therefore manufacturers always place such characteristics in the quality certificate of sensors intended for dynamic signals.

* Corresponding author.
E-mail addresses: k.tomczyk@cyfronet.pl (K. Tomczyk), elay@pk.edu.pl (E. Layer).

http://dx.doi.org/10.1016/j.measurement.2016.04.053
0263-2241/

The maximum value of the signal rate of change is equal to the maximum value of the sensor impulse response. This maximum value results from the derivative of sensor step response. The impulse response being the first derivative of the step response defines the velocity of the signal transferred by the sensor. Therefore the maximum value of this impulse response is equal to the maximum admissible velocity of our sensor [6].

In measurement theory, there are many criteria than can be used to define dynamic error. In the area of energetic the ISE is one of the most important. However, there are considerable problems with the analytical determination of signals maximizing this error. In the case of one constraint, referring only to the magnitude, analytical solutions are considered in detail in [6–8], while examples of their practical application are presented in [10]. Due to the necessity of solving the set of complicated integral-convolution equations, it is only possible to determine signals with up to about 25 switchings. For a higher number of switchings the equations presented in [6–8] can be too difficult to solve and in many cases the solution is even unattainable. For two constraints analytical solutions have not been yet found. In such a situation the problem of determining signals maximizing the ISE can be transferred to the heuristic approach. The good results give here the possibility of GA application, with the classical roulette method and execution of real coding of the parameters [11–14].

Until now, neither the value of the frequency component nor energy density of maximizing signals has been considered for cases of dynamic error criteria. Such analysis seems to be interesting in terms of evaluating the correctness the maximizing signals determined by means of GA. Taking into account that these signals are included in the class of the non-stationary signals, it seems reasonable to apply one of the tools for time–frequency analysis.

The CWT study using a scalogram based on Morlet wavelets [15–19] works well in such cases. It is worth noting that an application of the scalogram allows the detection of time-varying energy flux and transient bursts which usually are not easily detectable in the time or frequency domain [17].

As an example of the application of the CWT in the analysis of both the energy and the harmonic components of the signals maximizing ISE for the Althen731A and the Althen731-207 voltage output accelerometers is presented in this paper [20,21].

2. Theoretical principles of applied CTW analysis

The energy density of non-stationary signals $u(t)$, as a function of frequency most commonly represented by their wavelet scalograms. The scalogram presents square magnitude of the continuous wavelet transform CWT [15–17] and for $t \in (0, T)$ is defined as

$$Scal_u^{(\psi)}(s,\tau) = \left|CWT_u^{(\psi)}(s,\tau)\right|^2 = \left|\int_0^T u(t)\psi_{\tau,s}^*(t)d\tau\right|^2$$
$$= \frac{1}{\sqrt{s}}\left|\int_0^T u(t)\psi^*\left(\frac{t-\tau}{s}\right)d\tau\right|^2 \quad (1)$$

where $\psi_{\tau,s}^*(t)$ is the complex conjugate of the wavelet used for signal analysis.

The total time T refers to the signal time duration and corresponds to the steady state of the accelerometer impulse response.

In (1) the kernel

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \quad (2)$$

shifts and dilations in scale of the mother wavelet $\psi(t)$, $1/\sqrt{s}$ denotes the normalisation factor, s and τ are the scaling and the translation factors. The normalisation factor ensures the same energy for all wavelets.

The scaling s is a dimensionless parameter, while τ has a unit of time measurement [s].

The value of the signal energy is obtained indirectly, by double integration of the scalogram (1)

$$E_u = \int_0^T u(t)^2 dt = \frac{1}{C_\psi}\int_0^T \int_{s=0}^{+\infty} \frac{Scal_u^{(\psi)}(s,\tau)}{s^2} dtds \quad (3)$$

where

$$C_\psi = \int_0^\infty \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < \infty \quad (4)$$

is the admissible constant, and

$$\Psi(\omega) = \int_0^T \psi(t) e^{-i\omega t} dt \quad (5)$$

Eq. (3) represents the total energy density of the signal in the time-scale plane and is associated with the measurement of $\Delta s \frac{\Delta t}{s^2}$, where Δs and Δt are the scale and the time intervals.

In this paper the scalogram is based on the Morlet wavelet which can be presented in real or complex form.

The real form is

$$\psi^M(t) = e^{-t^2/2}\cos(5t) \quad (6)$$

and in most cases is used to sharpen signal transitions [16], while the complex form [18] is

$$\psi^M(t) = \frac{1}{\sqrt[4]{\pi}}\left[\exp(i\omega_0 t) - \exp(-\omega_0^2/2)\right]\exp(-t^2/2) \quad (7)$$

where ω_0 is the central frequency of the mother wavelet. It represents the centre of a Gaussian distribution of the particular frequencies in a Morlet wavelet.

The component $\exp(-\omega_0^2/2)$ in (7) is used to remove the non-zero mean value of the complex sinusoid and it can be neglected for $\omega_0 > 5$ [Hz]. Then, we have

$$\psi^M(t) = \frac{1}{\sqrt[4]{\pi}}\exp(i\omega_0 t)\exp(-t^2/2) \quad (8)$$

The particular wavelets corresponding to (8) are calculated based on (2) and are

$$\psi_{\tau,s}^M(t) =$$
$$= \tfrac{1}{s\sqrt[4]{\pi}}\exp\left[i\omega_0\left(\tfrac{t-\tau}{s}\right)\right]\exp\left[-\tfrac{1}{2}\left(\tfrac{t-\tau}{s}\right)^2\right] \quad (9)$$

Determining the Fourier transform of (9) we have

$$\Psi_{\tau,s}^M(\omega) == \frac{\sqrt{2\pi s}}{\sqrt[4]{\pi}}\exp\left[\frac{-(s\omega-\omega_0)^2}{2}\right]\exp(-i\omega\tau) \quad (10)$$

and the maximum of (10) gives the condition

$$\frac{\partial|\Psi_{\tau,s}^M(\omega)|}{\partial\omega} = 0 \quad (11)$$

After recalculation (11) the relation between the Morlet wavelet frequency ω_M and the scale s is obtained as

$$s = \frac{\omega_0}{\omega_M} = \frac{f_0}{f_M} \quad (12)$$

The maximum value of the scalogram gives a solution for the equation

$$\partial\frac{|CWT_u^{(\psi)}(s,\tau)|^2}{\partial s} = 0 \quad (13)$$

In MatLab, the scalogram is generated using the following function: *wscalogram('image',s,'scales' scales,'ydata',x, 'xdata',t)*, and *s = cwt(x,*