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Comparison of experimental observations in rotating machines with simple mathematical simulations

Adrian D. Nembhard, Jyoti K. Sinha *

The Dynamics Laboratory, School of Mechanical, Aerospace and Civil Engineering (MACE), The University of Manchester, Manchester M13 9PL, UK

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ABSTRACT

Through an earlier experimental study, an integrated fault classification approach was suggested for machines operating at different speeds. The suggested method was observed to separate faults adequately, which can lead to fault diagnosis. However, theoretical understanding of the proposed methods is important to further enhance the confidence of the earlier experimental study. Hence, simplified mathematical simulations of different rotor faults are carried out and used to test the earlier proposed classification technique in this paper. Observations in the orbit, spectra and fault classification diagrams are consistent with the previous experimental case.

1. Introduction

Vibration-based fault diagnosis (FD) of rotating machinery in industrial systems is a complex process which requires new or continuously improving techniques to meet the ever burgeoning demand for increased equipment reliability. In the development of new techniques, it may be useful to conduct theoretical simulations to test the utility of the approach and possibly to garner a greater appreciation of the dynamics of faults encountered. The Finite Element Method (FEM) is currently the staple method for the theoretical exploration of the dynamic effects of different faults on rotating machinery [1–6]. However, FEM is undoubtedly complex and there are instances, such as an initial theoretical investigation of a newly proposed FD technique, in which a simple mathematical model may be preferred.

The Jeffcott rotor is the simplest mathematical for understanding the dynamic behaviour of rotating machines under different conditions [7]. One of the earliest accounts of the use of the Jeffcott rotor for rotor fault investigation was provided by Jeffcott [8] who investigated the dynamics of an unbalanced shaft. Since then, it has been applied in numerous vibration-related studies with literature accounting extensively for studies on cracks and rotor-stator rub and to a lesser extent other faults.

Jun et al. [9] applied a Jeffcott rotor to identify the characteristics of a breathing crack which opens and closes. Using fracture mechanics it was concluded that the switching crack model with bi-level direct stiffness was mathematically simple but physically sensible when the rotor operated well below the second bending critical speed. Gasch [10] studied the dynamic behaviour of a transverse crack on a Laval rotor that was modelled like a low pressure turbine with weight dominance. Time dependent stiffness variations were based on the hinge model. It was suggested that trend analysis from permanent monitoring would prove helpful to early crack detection. In contrast, Cheng et al. [11] analysed the dynamic response of a Jeffcott rotor with a breathing crack sans the typical weight dominance assumption. Simulations were run near the system's critical speed and shaft breathing was expressed a function of the whirling of the rotor. It was observed that the response of a cracked rotor near its critical speed is similar to those of a healthy rotor. Patel and Darpe [12] cautioned the use of the switching crack model to predict the vibration response of a cracked rotor with deeper cracks. It was found that unbalance phase, unbalance level, crack depth and system damping have significant effect on the vibration response of the rotor with switching crack but have no influence on the rotor with breathing crack. However, Penny and Friswell [13] provided an insightful comparison of the vibration response of a simple Jeffcott rotor with three different crack models, namely; the hinge model, the Mayes model and the model of Jun et al. and suggested that the differences in the models would be negligible in a crack identification scheme.

Chu and Zhang [14] employed a Jeffcott rotor to examine the nature of rotor to stator rub. Equations of motions of the system were developed and solved via a Runge Kutta integration scheme. It was observed that the rub feature transitioned to three different

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* Corresponding author.
E-mail addresses: adrian.nembhard@manchester.ac.uk (A.D. Nembhard), jyoti.sinha@manchester.ac.uk (J.K. Sinha).

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Nomenclature

a	crack depth
c	damping coefficient of shaft
d	rotor radial displacement
d_0	rotor-stator radial clearance
e_a	added mass eccentricity
e_d	disc eccentricity
E	modulus of elasticity
F_{nC}	rub normal contact force
F_{tC}	rub tangential friction force
F_x, F_y	general forces on rotor
F_{xM}, F_{yM}	misalignment reaction forces on rotor (x and y)
F_{xR}, F_{yR}	rub forces on rotor (x and y)
FX, FY, FZ	coupling reaction forces
g	acceleration due to gravity
k	rotor (shaft) stiffness
k_C	stator stiffness
K_b	coupling bending spring rate
m	rotor mass
m_a	added mass
MX, MY, MZ	coupling moments due to misalignment
P	motor power
r_s	shaft radius
R	disc radius
t	time
Tq	drive torque
u, v	rotor displacement (x and y)
$\dot{u}$, $\dot{v}$	rotor velocity (x and y)
$\ddot{u}$, $\ddot{v}$	rotor acceleration (x and y)
u', v'	rotor displacement at 45° (x and y)
x, y	coordinate directions
$Z1$, $Z2$	shaft centreline (driver and driven)
$Z3$	coupling centre of articulation
α	added mass phase angle
β	unbalance phase angle
γ	misalignment phase angle
ζ	damping ratio
μ	friction coefficient
ρ	rotor density
τ	rotor-stator stiffness ratio
ψ	velocity at rotor-stator contact point
ω	angular velocity
ω_n	natural frequency
$\Delta X, \Delta Y$	misalignment offset (x and y)

routes to chaos on increasing rotating speeds. Similarly, Lin et al. [15] was able to study the behaviour of rub-related vibrations with numerical simulations on a Jeffcott rotor model in order to "reveal the nonlinear mechanisms" underlying the chaotic phenomena previously observed in works such as Chu and Zhang [14]. Though simplified, the model was able to capture the rich dynamics of a rotor system rubbing its housing. It was found that there exist threshold values for rotor-stator clearance and friction coefficient above which the bouncing phenomena would not be experienced. It was also suggested that the results of the simulations from such a simple model can be useful for the development of sensitive machinery diagnostic techniques.

Just over a decade ago, Redmond and Al-Hussain [16] and Al-Hussain and Redmond [17] bemoaned the lack of requisite theoretical studies with thorough treatment of the nature of misalignment using "suitably representative models". Consequently, a coupled twin Jeffcott rotor model was similarly offered by Redmond and Al-Hussain [16] and Al-Hussain and Redmond [17] to provide insight to the dynamic behaviour of machines with coupling misalignment and residual unbalance. The former [16] explored a flexible coupled rotor model, while the latter [17] investigated a rigidly coupled set up. Both studies considered only pure parallel misalignment and solved the system's equations with the Newmark-Beta method of integration. Redmond and Al-Hussain [16] found that the presence of misalignment caused a radial preload in the rotor system which modified the system vibration amplitude and phase characteristics. More interestingly, Al-Hussain and Redmond [17] did observe the customary 2X harmonic component in the rigidly coupled system response, therefore leading to the conclusion that other system properties including rotor asymmetry and in some cases coupling kinematics are the sources of higher harmonics in misalignment response.

Thought it is opined that the Jeffcott model is too simple and limited to a qualitative understanding of the machine state which does not concur to "real life conditions" [5], the literature reviewed here made observations somewhat contrary to such claims; as the Jeffcott rotor model enabled adequate representation of the dynamic conditions of rotating machine under different conditions. In light of this, the Jeffcott rotor is used in the current study to generate responses of commonly encountered rotor related faults in order to further test the performance of an earlier developed vibration-based FD approach [18] with theoretical data. In the earlier work, the FD method was developed in the Dynamics Lab at the University of Manchester on a small experimental rig. It was intended that modelling of the Jeffcott rotor would preserve the simplicity of the simulation process while yet still producing responses representative of real data.

Five different rotor conditions were simulated in the current study namely; low unbalance, added unbalance, crack, misalignment and rub. To solve the equations of motion developed for the different models the Newmark beta integration scheme [19] is employed. Orbit plots and simple amplitude spectra were compared to check the accuracy of the models. The stead-state time domain response from each model were used to compute condition indicators which were then input to the two classification techniques being tested here; single speed (SS) and multi speed (MS) analysis. The results of classification with both SS and MS analyses had good separation of the conditions tested with the latter having slightly improved separation. The observations made in the current study were consistent with the fault classification done in the earlier experimental study, which suggest the methods perform regardless of data source. The findings made here bode well for the assertion of these vibration-based FD methods as viable.

A brief description of the layout of the paper follows. In Section 2 a brief description of the earlier experimental study including a review of the classification technique is provided for reference purposes. Afterwards, in Section 3, the simple model of the Jeffcott rotor is derived. Subsequent to this, detailed descriptions of the simulation of different rotor conditions tested are provided in Section 4. Section 5, presents fault classification with the simulated data in the earlier developed technique. Finally, conclusions are drawn in Section 6.

2. Earlier experimental study [18]

In this section, the previously used experimental rig is presented and experiments done are highlighted. Then, to offer some perspective to the dynamic effects of the different faults tested on the experimental rig, orbit pots representative of the different