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Improved discrete Fourier transform algorithm for harmonic analysis of rotor system

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ABSTRACT

Harmonic components are critically important to the fault diagnosis of rotor system. Various methods have been developed for harmonic component extraction. The challenge however, is to efficiently and accurately extract rotor system harmonic components. In this paper, a new harmonic analysis approach based on an improved discrete Fourier transform algorithm is proposed. It schemes out harmonic analysis sampling and resampling method to obtain synchronous sampling data, which can realize integer-period synchronous sampling and greatly increase the measurement accuracy of harmonic parameters. Then an improved discrete Fourier transform algorithm is proposed to extract harmonic components, which can dramatically reduce the computation load through replacing multiplication by shift operation. Finally, the effectiveness of the proposed method is analyzed by means of simulation and practical experiment for multifrequency simulated signal and shaft misalignment faults, respectively. Results show that the proposed method is faster and more accurate than both the DFT-based methods and the FFT-based methods for extracting harmonic components of rotor system.

1. Introduction

Rotating machinery is the most widely used equipment in industry. As the core part of rotating machinery, rotor system often appears vibration faults, which would affect the normal work of rotating machinery and cause substantial economic losses, even personal injury and death. Common vibration faults include rotor unbalance, shaft misalignment, broken tooth, badly lubricated slide bearing, etc., all of which can be diagnosed through harmonic analysis [1,2]. Thus, study on harmonic analysis of rotor system to extract harmonic components is of great necessity. The existing harmonic analysis methods utilize a variety of techniques, such as discrete Fourier transform (DFT), the least square method [3], non parametric and parametric methods [4], harmonic balance method [5,6], wavelet analysis [7,8], Hilbert–Huang transform [9], artificial neural network [10]. Thereinto, DFT can be regarded as a special case of the discrete-time Fourier transform, and is suitable for separate frequency analysis, but its computation speed is slow and measurement accuracy is affected by asynchronous sampling, non-integer-period truncation and interharmonics. Parameterize method has very high time–frequency resolution, but due to huge computation load, its model parameters are hard to decide. Harmonic balance method is capable of extracting the harmonic component to any desired accuracy, but needs to optimize numerous variables with a large number of computation operations. Wavelet analysis has the advantage of detecting the fast and frequent fluctuations of harmonics but sensitive to noise. Hilbert–Huang transform provides a

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new method of analyzing nonstationary and nonlinear signal, but it is more like an empirical approach rather than a theoretical tool and needs to be perfected. Artificial neural network enables independent study during application, but its initial training samples are hard to decide. Besides, when real-time performances are required, these methods cannot give a satisfactory outcome. In these instances, as a fast algorithm of DFT, the fast Fourier transform (FFT) is preferable for its availability, understandability, simplicity, rapidness, and easiness to implement [11]. However, like the DFT-based methods, the FFT-based methods under asynchronous sampling suffer from the drawbacks of spectrum leakage and barrier effect [12,13], which affect the accuracy of harmonic measurement. In order to overcome these problems, a number of improved methods based on DFT algorithm or FFT algorithm have been proposed, e.g. multipoint interpolation DFT [14], recursive sliding DFT [15], adjustable windowed FFT [16], Hanning self-convolution windowed FFT [17], Flat-top windowed FFT [18] and All-phase FFT [19]. Although to some extent, these methods can improve the accuracy of harmonic measurement, they still cannot solve these problems completely due to non-integer-period truncation and interharmonics.

Considering rotor system harmonic analysis mainly focuses on several specific harmonic frequency points, the question of whether there is an algorithm that aims at harmonic component extraction and can extract harmonic components more efficiently and accurately than both the DFT-based methods and the FFT-based methods is presented. In view of this, a rotor system harmonic analysis method based on an improved discrete Fourier transform (IDFT) algorithm is proposed in this paper. Firstly, the sampling and resampling formulas of the IDFT-oriented harmonic analysis are given to obtain sampling data, which can ensure that the obtained data are sampled with integer-period and no spectrum leakage. Then, based on the DFT algorithm, the IDFT algorithm with better performance is proposed to extract the harmonic parameters of rotor system. The proposed method has shorter computation time by using shift operations and can accurately extract harmonic parameters at specific frequency points.

The remainder of the paper is organized as follows: The IDFT-oriented harmonic analysis sampling and resampling method is deeply developed in Section 2. Section 3 gives a detailed discussion on the IDFT algorithm principles on the basis of using the DFT algorithm to deduct the IDFT algorithm equation, and then provides a process for the realization of the IDFT numerical algorithm. Section 4 validates the feasibility of IDFT-based rotor system harmonic analysis through simulation and experiment. Finally, conclusions are drawn in Section 5.

2. The IDFT-oriented harmonic analysis sampling and resampling method

The IDFT-oriented harmonic analysis sampling and resampling method provides the basis for implementation of the IDFT algorithm, which is consisted of IDFT-oriented harmonic analysis sampling method and IDFT-oriented harmonic analysis resampling method. Before discussing the IDFT algorithm, we will demonstrate IDFT-oriented harmonic analysis sampling method and IDFT-oriented harmonic analysis resampling method.

To demonstrate the IDFT-oriented harmonic analysis sampling and resampling method, we take the process of sampling a continuous-time signal $x(t)$, which includes I integer harmonics, as an example. The time-domain representation of the signal $x(t)$ is

$$x(t) = \sum_{i=1}^{I} A_i \cos(2\pi f_i t + \varphi_i) + y(t), \text{ for } t \geqslant 0, \ f_i = i f_1 \quad (1)$$

where i is the order of a harmonic in the signal $x(t)$, I is the total number of harmonic components in the signal $x(t)$, f_1 is the fundamental frequency of the signal $x(t)$, f_i, A_i and φ_i are frequency, amplitude and phase of the ith harmonic in the signal $x(t)$, respectively, and $y(t)$ is the residual part of the signal $x(t)$ except the integer harmonics.

2.1. The IDFT-oriented harmonic analysis sampling method

Definition 1. To assure the synchronous sampling data can be collected, the IDFT-oriented sampling frequency f_s is defined

$$f_s = [a_1, a_2, \ldots a_I] 12 f_1 \quad (2)$$

where $a_1, a_2, \ldots a_I$ for $a_1 < a_2 < \cdots < a_I$ correspond to all the harmonic orders of the signal $x(t)$, and $[a_1, a_2, \ldots a_I]$ represents the lowest common multiple of $a_1, a_2, \ldots a_I$.

Analysis of (2) reveals that the IDFT-oriented sampling frequency f_s is defined by two characteristics: (I) $[a_1, a_2, \ldots a_I]$ times of the fundamental frequency f_1; and (II) twelve integer times of each harmonic frequency. The former characteristic can assure that synchronous sampling data can be collected for extraction of each harmonic component and no resampling error is imported into subsequent resampling processing, because the IDFT-oriented sampling frequency f_s is set integer times of each harmonic frequency. The latter characteristic can assure that the IDFT-oriented sampling frequency f_s meets Nyquist–Shannon sampling theorem and the base-2 property depicted in (16) (which will be introduced later) is not lost.

In engineering application, although f_s may be significantly higher than the frequency required by Nyquist–Shannon sampling theorem, f_s will not be too high, for the reason that the fundamental frequency f_1 keeps relatively small in the field of mechanical (typically several tens or hundreds of kHz), and that the harmonic order a_I should not be too high, considering the total number of harmonic I being meaningful to rotor system fault diagnosis is limited. Thus, the defined IDFT-oriented sampling frequency f_s has certain engineering application value.

Definition 2. To ensure the analyzed data are sampled with integer-period, the IDFT-oriented sampling length N is defined