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On the actual and observed process capability indices: A signal-to-noise ratio model

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ABSTRACT

In the evaluation of process capability, gauge measurement errors usually distort the measured data yielding two dissimilar capability indices, particularly, the actual and the observed process capability indices (AC_p and OC_p). Gauge measurement errors result in underestimation of the actual process capability, consequently, the variance of gauge errors has to be assessed to better chart the relationship between the AC_p and OC_p. The different variance components of a measurement system can be assessed by a gauge repeatability and reproducibility (GR&R) study. This paper presents novel relationships between the AC_p and OC_p by means of a signal-to-noise ratio (*SNR*) model. The probability density functions of both indices will be presented in terms of *SNR* and a procedure to find the critical values of AC_p and OC_p is established. In contrast to literature studies, a measurement system can now be described by a novel α–β characteristic curve. Different *SNR* values will result in different α–β curves, hence, the acceptance of a measurement system depends on the specified significance values of α and β and not solely on strict *SNR* values. Since measured data yields two different AC_p and OC_p distributions, type I and type II error analysis can be performed. Different case studies are presented to validate the resulting relationships and distributions.

1. Literature review

Organizations thrive to improve their product quality, a matter which necessitates a frequent revision of their measurement systems for up-to-standard and consistent production. Accordingly, a gauge repeatability and reproducibility (GR&R) study is recommended to assess the adequacy of a measurement system. GR&R is performed according to the MSA handbook stated in QS9000 standards, [1]. To avoid underestimating the actual process capabilities, a GR&R study needs to be conducted prior to the process capability analysis [12,5,21,20].

Other methods used to analyze gauge variation include Principle Component Analysis (*PCA*) (Osma [10], Wang [25], Peruchi et al. [18], Peruchi et al. [19]). As the “unit of measure” for the quality characteristics will disappear after conducting *PCA*, such methods may not point out to the causes of production problems [13].

Currently, there are different measures used to judge the adequacy of a measurement system such as the Precision-to-Tolerance ratio (*PTR*), the Signal-To-Noise ratio (*SNR*) and the discrimination ratio (*DR*). Such ratios are described in many well known references as Jheng [6], Burdick et al. [3], Pan [11], Pearn et al. [17] and Pan and Huang [13]. Indeed, there is a quite interesting research addressing process capability indices under the consideration of the acceptable ranges of the above ratios, [8,16,7,22]). One major concern in most of these studies

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was the exclusion of measurement errors that may lead to underestimation of the true capability indices, [4,26].

In this paper, new measurement guidelines are introduced that consider critical values applied to both the actual and observed capability distributions. The probability distributions of the AC_p and OC_p will be formulated as a function of the *SNR*. A new hypothesis test is introduced to benchmark the AC_p and OC_p yielding a new measurement characteristic α–β curve as compared to strict *PTR* or *SNR* thresholds. By means of both the AC_p and OC_p distributions, the chances of evaluating the process capability under an erroneous/error free gauge will be found as type I and type II errors.

The sections of this paper are organized follows: Section 2 presents an introduction to the measurement systems and GR&R study. Section 3 demonstrates the hypothesis and the χ^2 property along with the actual and observed capability indices, followed by Section 4 which explains the process capability domains. Later in Section 5, the *SNR* model is laid out followed by the measurement system characteristic curve in Section 6. The novel C_p distribution is found in Section 7. Next in Section 8, supplementary demonstrations are depicted in a number of figures to resolve the tradeoff between the two capability indices followed by the experimental results in Section 9 and finally the conclusion in Section 10.

2. Introduction

The process capability index is a quantitative measure of the ability of a process to meet predetermined specifications. The process capability index is given by:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

where *USL* and *LSL* are the upper and lower specification limits and σ is the standard deviation. However, measurement errors may exist in the data used to assess the process capabilities, which have an adverse effect on the resulting variance.

Several quality measures are used to assess the adequacy of a measurement system such as the Precision-to-Tolerance ratio (*PTR*) and Signal-to-noise ratio (*SNR*). Usually, if $PTR < 0.1$ then the gauge is said to be capable, while if $PTR > 0.3$ then the gauge is not capable. The gauge is moderately capable if the *PTR* lies between those two thresholds, [9].

Similarly, if $SNR > 5$, the gauge is considered as capable, while if $SNR < 2$ the gauge is considered as incapable, while the *SNR* between 2 and 5 indicates moderately acceptable gauge [15,3,2]. However, strict thresholds do not precisely judge the gauge, as different measurement capabilities may result even with one specific *SNR* value as will be illustrated shortly. Due to the additive gauge variability, the observed process capability will be less than the actual (true) process capability.

In measurement systems, the different sources of variability can be identified using GR&R study. The variances included in GR&R study are: $\hat{\sigma}^2_{repeatability}$ and $\hat{\sigma}^2_{Reproducability}$. Repeatability is the ability of an operator to consistently repeat the same measurement of the same part, using the same gauge, under the same conditions. On the other hand, reproducibility is the ability of a gauge, used by multiple operators, to consistently reproduce the same measurement of the same part, under the same conditions. Hence, the gauge variance is given by:

$$\hat{\sigma}^2_g = \hat{\sigma}^2_{repeatability} + \hat{\sigma}^2_{Reproducability} \tag{2}$$

The total measurement system variance is estimated by the addition of the gauge variance $\hat{\sigma}^2_g$ to the part variance $\hat{\sigma}^2_p$, [5,9]. That is:

$$\hat{\sigma}^2_{total} = \hat{\sigma}^2_p + \hat{\sigma}^2_g \tag{3}$$

In our analysis, the *SNR* will be used to establish the new AC_p and the OC_p distributions. The signal-to-noise ratio of a measurement system is found in Montgomery [9]:

$$SNR = \frac{\sqrt{2}\sigma_p}{\sigma_g} \tag{4}$$

3. The χ^2 property of the actual and observed process capabilities

Let c represent a benchmark value of the process capability index (*PCI*). A simple statistical test can be established as follows:

$$H_0 : PCI \leqslant c,$$

$$H_1 : PCI > c.$$

where the null hypothesis states that the process does not satisfy the quality requirement. Recall that the ratio between the estimated and true variance follows the χ^2 distribution, [12]:

$$(n-1)\left(\frac{\hat{\sigma}_{total}}{\sigma_{total}}\right)^2 \sim \chi^2_{n-1}, \tag{5}$$

Hence, we can easily find that:

$$\frac{C_p}{\hat{C}_p} = \frac{\hat{\sigma}_{total}}{\sigma_{total}} \tag{6}$$

Plugging (6) into (5), we get:

$$(n-1)\left(\frac{C_p}{\hat{C}_p}\right)^2 \sim \chi^2_{n-1}, \tag{7}$$

The above form has been repeatedly verified in the literature [12,14]. Using the above χ^2 distribution, a critical value can be set at α level of significance in the χ^2 domain as:

$$\alpha = P(\hat{C}_p > c_0 | C_p = c) = P\left(\chi^2 < \frac{(n-1)c^2}{c_0^2}\middle| C_p = c\right)$$

Consequently, we get the expression $\frac{(n-1)c^2}{c_0^2} = \chi^2_{1-\alpha,n-1}$, where $\chi^2_{1-\alpha,n-1}$ represents the χ^2 value at the lower $1-\alpha$ quantile for $(n-1)$ degrees of freedom, hence, the ratio between the critical value and the C_p can be put across as: