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Robust Nonlinear Attitude Stabilization of a Spacecraft through Digital Implementation of an Immersion & Invariance Stabilizer

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Abstract: The paper deals with the problem of robust nonlinear attitude stabilization of a rigid spacecraft. In particular, an Immersion and Invariance robust attitude stabilizer is proposed, taking into account actuator dynamics in control design. The proposed continuous-time controller is then implemented under sampling using an approximated single-rate strategy to match, at the sampling instants, the zero-going evolution of the off-the-manifold coordinates. Simulations show the effectiveness of the proposed controller.

Keywords: Robust nonlinear control, Immersion and Invariance, Sampled-data control.

1. INTRODUCTION

In this paper, a nonlinear control strategy to stabilize the attitude of a rigid spacecraft robustly with respect to actuators dynamics is proposed. The control law is applied under sampling using a single-rate digital approach with first order corrector term, which overcomes in performance the direct implementation through zero-order hold, as shown by simulations. A robust attitude stabilizer is necessary for long range communications satellites, especially when a high throughput is involved. The capability of the spacecraft to maintain a fixed orientation despite external disturbances and modeling uncertainties is crucial when dealing with satellite internet access at high data speeds (Pietrabissa and Fiaschetti (2012)).

The technique of Immersion & Invariance (I&I), first introduced by Astolfi and Ortega (2003), is a tool for the stabilization of nonlinear systems via state-feedback. The existence of a reduced globally asymptotically stable dynamics, called target dynamics, which can be immersed into the system to be controlled, plus the invariance and attractivity of the corresponding manifold, together with the boundedness of the trajectories of an extended system are sufficient conditions for the GAS of a chosen equilibrium of the controlled system. The method is specifically suitable for systems that admit a fast-slow dynamics decomposition, e.g. singularly perturbed systems, systems in feedback form but also underactuated systems requiring non-standard control solutions (Astolfi et al. (2008)). Applications to spacecraft have been proposed mostly in the adaptive context in presence of flexible dynamics, as in Lee and Singh (2009). In the sampled-data context, a multi-rate implementation of an I&I stabilizer is introduced for a class of feedback systems in Mattei et al. (2014). Nonlinear sampled-data controllers are also developed to solve the attitude tracking problem in Di Gennaro et al. (1999) and in the flexible case in Monaco et al. (1986).

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In this work we propose an I&I solution for systems in strict-feedback form which is particularly suited to counteract the degrading effect of unmodeled actuator dynamics on the overall control systems. In fact, I&I can be regarded as a tool to robustify a given nonlinear controller with respect to higher-order dynamics, exploiting at its best the knowledge of such dynamics during the control design phase. Thus, this approach can be considered "robust" nonlinear control. The obtained continuous-time controller is then implemented under sampling using a single-rate control strategy with truncation of series expansions at the second order in the sampling period. Simulations at increasing sampling times show the effectiveness of using a first order corrector term with respect to the simpler implementation through zero-order hold device (emulated control Nesic et al. (1999)). In particular, the maximum allowable sampling period (MASP) is increased, thus the sampled-data controller shows robustness w.r.t. δ, sampling time.

Notations All the functions, maps and vector fields are assumed smooth and the associated dynamics forward complete. Given a vector field f, L_f denotes the associated Lie derivative operator, $L_f = \sum_{i=1}^{n} f_i(\cdot)\frac{\partial}{\partial x_i}$, e^{L_f} denotes the associated Lie series operator, $e^f := 1 + \sum_{i\geq 1} \frac{L_f^i}{i!}$. For any smooth real valued function h, the following result holds $e^f h(x) = e^f h|_x = h(e^f x)$ where $e^f x$ stands for $e^{L_f} I_d|x$ with I_d the identity function on $\mathbb{R}^n$ and (x) (or equivalently $|_x$) denotes the evaluation at a point x of a generic map. The evaluation of a function at time $t = k\delta$, δ sampling period, indicated by "$|_{t=k\delta}$" is omitted, when it is obvious from the context. The notation $O(\delta^p)$ indicates that the absolute value of the approximation error in the series expansions is bounded from above by a linear function of $|\delta|^p$, for δ small enough. With $\mathrm{col}(x_1, x_2, \ldots, x_n)$ we denote the column vector of components $x_1, x_2, \ldots, x_n$. A class $\mathscr{K}$ function is a continuous function $w(\cdot) : [0,a) \to [0,\infty)$, which is strictly increasing and

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such that $w(0) = 0$. A class $\mathcal{K}_\infty$ function is a class $\mathcal{K}$ function with $a = \infty$ and such that $\lim_{r\to\infty} w(r) = \infty$. Throughout the paper, the gradient of a scalar function is considered a row vector.

2. SPACECRAFT DYNAMIC MODELING

Consider a symmetric rigid spacecraft characterized by a diagonal inertia matrix J, namely

$$J = \begin{pmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{pmatrix}.$$

The kinematic model used is based on the *modified Cayley-Rodrigues parameters*, which provide a global and non-redundant parametrization of the attitude of a rigid body (Dwyer et al. (1987)). In the following, $S(\cdot)$ denotes the three-dimensional skew-symmetric matrix, which for a generic vector $r \in \mathbb{R}^3$ takes the form

$$S(r) = \begin{pmatrix} 0 & r_3 & -r_2 \\ -r_3 & 0 & r_1 \\ r_2 & -r_1 & 0 \end{pmatrix} \tag{1}$$

Defining $\rho \in \mathbb{R}^3$ the modified Cayley-Rodrigues parameters vector and $\omega \in \mathbb{R}^3$ the angular velocity in a body-fixed frame, the kinematic equations take the form

$$\dot{\rho} = H(\rho)\omega. \tag{2}$$

The matrix-valued function $H : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ denotes the kinematic jacobian matrix of the modified Cayley-Rodrigues parameters, given by

$$H(\rho) = \frac{1}{2}\left(I - S(\rho) + \rho\rho^T - \frac{1+\rho^T\rho}{2} I\right) \tag{3}$$

where I denotes the 3×3 identity matrix. The matrix $H(\rho)$ satisfies the following identity (Tsiotras (1996))

$$\rho^T H(\rho)\omega = \left(\frac{1+\rho^T\rho}{4}\right)\rho^T\omega \tag{4}$$

for all $\rho, \omega \in \mathbb{R}^3$.

According to Euler's law, the kinematic and dynamic equations can be written as

$$\dot{\rho} = H(\rho)\omega \tag{5}$$

$$\dot{\omega} = J^{-1}S(\omega)J\omega + J^{-1}u. \tag{6}$$

If first-order actuator dynamics with time-constants T_i ($i = 1,2,3$) are considered, equations (5)-(6) are dynamically extended as follows

$$\dot{\rho} = H(\rho)\omega \tag{7}$$

$$\dot{\omega} = J^{-1}S(\omega)J\omega + J^{-1}\tau \tag{8}$$

$$\dot{\tau} = A\tau + u. \tag{9}$$

where $\tau \in \mathbb{R}^3$ represents the torque generated by the actuators according to the reference torque $u \in \mathbb{R}^3$ and $A = \mathrm{diag}(-\frac{1}{T_1}, -\frac{1}{T_2}, -\frac{1}{T_3})$ is a Hurwitz diagonal matrix whose eigenvalues, all negative real, depend on the time constants of the actuators.

3. IMMERSION AND INVARIANCE STABILIZATION

3.1 Recalls

Let us recall the continuous-time I&I main result in the general case (the proof is detailed in Astolfi et al. (2008)).

Theorem 1. Consider the nonlinear system

$$\dot{x} = f(x) + g(x)u \tag{10}$$

with state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}^m$ and an equilibrium point $x^* \in \mathbb{R}^n$ to be stabilized. Suppose that (10) satisfies the following four conditions.

H1c (*Target System*) - There exist maps $\alpha(\cdot) : \mathbb{R}^p \to \mathbb{R}^p$ and $\pi(\cdot) : \mathbb{R}^p \to \mathbb{R}^n$ such that the sub-system $\dot{\xi} = \alpha(\xi)$ with state $\xi \in \mathbb{R}^p$, $p < n$, has a (globally) asymptotically stable equilibrium at $\xi^* \in \mathbb{R}^p$ and $x^* = \pi(\xi^*)$.

H2c (*Immersion condition*) - For all $\xi \in \mathbb{R}^p$, there exists a map $c(\cdot) : \mathbb{R}^p \to \mathbb{R}^m$ such that

$$f(\pi(\xi)) + g(\pi(\xi))c(\xi) = \frac{\partial\pi}{\partial\xi}(\xi)\alpha(\xi) \tag{11}$$

H3c (*Implicit manifold - $\mathcal{M}$*) - There exists a map $\phi(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n-p}$ such that the identity between sets $\{x \in \mathbb{R}^n | \phi(x) = 0\} = \{x \in \mathbb{R}^n | x = \pi(\xi) \text{ for } \xi \in \mathbb{R}^p\}$ holds.

H4c (*Manifold attractivity and trajectory boundedness*) - There exists a map $\psi(\cdot,\cdot) : \mathbb{R}^{n\times(n-p)} \to \mathbb{R}^m$ such that all the trajectories of the system

$$\dot{z} = \frac{\partial\phi}{\partial x}[f(x) + g(x)\psi(x,z)] \tag{12a}$$

$$\dot{x} = f(x) + g(x)\psi(x,z) \tag{12b}$$

are bounded and satisfy $\lim_{t\to\infty} z(t) = 0$.

Under these four conditions, x^* is a globally asymptotically stable equilibrium of the closed-loop system

$$\dot{x} = f(x) + g(x)\psi(x,\phi(x)) \tag{13}$$

The following definition is straightforward.

Definition 3.1. (**I&I Stabilizability**). A nonlinear system of the form (10) is said to be *I&I stabilizable* with target dynamics $\dot{\xi} = \alpha(\xi)$, if it satisfies conditions H1c to H4c of Theorem 1.

Note that the target dynamics is the restriction of the closed-loop system to the manifold $\mathcal{M}$, implicitly defined in H3c. The control law $u = \psi(x,z)$ is designed to steer to zero the off-the-manifold coordinate z and to guarantee the boundedness of system trajectories. On the manifold, the control law is reduced to $\psi(\pi(\xi),0) = c(\xi)$, and it renders $\mathcal{M}$ invariant according to H2c. The complete control law can thus be decomposed in two parts:

$$u = \psi(x,\phi(x)) = \psi(x,0) + \tilde{\psi}(x,\phi(x)) \tag{14}$$

with $\psi(\pi(\xi),0) = c(\xi)$ on the manifold and $\tilde{\psi}(x,0) = 0$. Note that $\psi(x,0)$ can be seen as a nominal control law, designed on the model of the dynamics restricted on the manifold to obtain a GAS target dynamics. In this sense, the term $\tilde{\psi}(x,\phi(x))$ is a robustness-improving addendum which takes into account the off-the-manifold behaviors generated, for instance, by higher-order actuator dynamics. The overall control law provides the I&I "robust" nonlinear stabilizer.

3.2 The class of systems under study

In this work, we consider the problem of state-feedback stabilization of the following class of systems in feedback form

$$\begin{aligned} \dot{\xi} &= f(\xi) + g(\xi)\eta \\ \dot{\eta} &= u \end{aligned} \tag{15}$$

where $\xi \in \mathbb{R}^p$, $\eta \in \mathbb{R}^{n-p}$, $u \in \mathbb{R}^m$ (with $m = n-p$), $x = \mathrm{col}(\xi,\eta)$ and $\xi = 0$ is a globally asymptotically stable equi-