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A first-order probabilistic logic with application to measurement representations

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ABSTRACT

A first order probabilistic logic is developed and presented in both intuitive and formal terms. It is shown how it can be successfully applied to the development of probabilistic representations for the main structures and scales involved in (one-dimensional) measurement. As a part of the current debate on the nature of probability in measurement, this result provides a way for overcoming the traditional opposition between Bayesian and orthodox statistics.

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1. The question of the nature of probability in measurement

The question of the nature of probability has been a part of the epistemological debate of the nineteenth century [37]. According to Hacking [11,27],[1] two main views emerged, where probability was understood either as expressing the *relative frequency* (of an *event*) or as *a degree of belief* (about a *statement*). Although these views are often compatible – for example, if we consider the probability that tomorrow in Genoa it will rain, we can regard "tomorrow in Genoa it will rain" either as an event that *may happen or not*, or as a statement that *may be true or false*, and things work in both cases – differences there are, as we will see in the following.

The measurement scientific community was also partially involved in this debate in regard with the discussion on uncertainty that paralleled [6,23] and followed [29,39,40] the development of the Guide to expression of uncertainty (GUM for short, [24]).

The challenge in those years was to attain a common and agreed way of accompanying measurement results with a statement on the uncertainty associated to them in virtually all the fields in which they are used. For doing so it was necessary to go beyond the classical theory of errors, since the absence of systematic errors postulated in that theory could not be assumed in the general case.

Now, whilst the interpretation of probability as a relative frequency is compatible with the classical error theory, it becomes critical when dealing with systematic effects, that in that theory are assumed to be negligible. So, although the GUM does not make a choice between the two interpretations, that are both included in an Appendix of its, it was argued by some authors that the degree-of-belief interpretation should be adopted and, since such interpretation is popular especially in the so called Bayesian statistics, it was hold that this latter approach should also be adopted [23,40]. Perhaps, in our opinion, the attitude of adopting results from another discipline – namely, statistics – prevailed in respect of the effort of developing original solutions.

In fact, Bayesian statistics [19,33] is not an homogeneous school of thought. For example one of its mainstreams pushes the degree-of-belief interpretation to its extreme consequences, yielding a subjectivist attitude [10]: probability ultimately expresses the standpoint of a single person! It goes without saying that such a position is hardly tenable in science, where objectivity is a major

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[1] References are listed in chronological order to give a feeling of the historical development of the subject.

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goal. Measurement in particular is expected to be an objective way of gathering information from the real world [16] and the result of measurement being expressed in a subjective way would sound strange. In view of these difficulties, other forms of the Bayesian approach have developed, such as the so called "objective Bayesian" one [33,42].[2]

Yet there is another fact to be considered. If measurement is intended – as usually is – as a universal tool for science, technology and everyday life, we should not forget that many, probably the majority of, professional statisticians use mainly (or exclusively) methods and tools from "orthodox",[3] not Bayesian, statistics [5,21,41]. Therefore, establishing a tight link between measurement and Bayesian statistics would create an evident conflict in many important application areas. In this regard, wise was the GUM, in our opinion, in not making an explicit choice.

Luckily, it is possible to develop a coherent probabilistic measurement theory without making any explicit commitment to Bayesian statistics, although using when needed the Bayes–Laplace rule, which in itself is just a probability calculus rule and, as such, does not require any philosophical commitment [36,44,47].

How can then probability be understood in such an approach?

One possibility is to simply regard probability as *a primitive notion*, mathematically defined by a set of axioms. This is basically referable to the axiomatic approach inaugurated by Kolmogorov [3] and currently adopted in many, probably most of, textbooks on probability theory [34].

Yet another suggestive possibility is to regard probability as *a logic*: this is what we will attempt in this paper, at least at an introductory level [46].

2. Probability as a logic: historical background

The idea of interpreting probability as a logic is not new and it is has been pursued to some extent in the past [17,33]. In a loose sense it is related to having an *epistemic* vision of probability, that is seeing it as *describing our state of knowledge* rather than *the way things actually behave in the outside world*, which would correspond to an *ontic* vision [33].

In this loose sense, Laplace himself can be counted among the supporter of a logical understanding of probability, since he had a deterministic vision of the world – a famous passage of his on the Supreme Intelligence is considered a manifesto of determinism – and justified the need of using probability with the weakness of our intelligence [33]. Boole also introduced probability in the context of his revision of logic [1,2], and Keynes also had a similar attitude [33]. But for a formal theory of probability as a logic we have to wait for Carnap [4] and his school [7,15]. He pursued these studies as a part of his ambitious programme of providing a logical foundation to the overall framework of science, and *logicistic* this overall approach is called [27]. His approach was strongly connoted by two main choices:

- He, as De Finetti, regarded probability as an essentially bi-argumental function, that is he held – in opposition to Kolmogorov's axiomatisation – that relative (conditional) probability comes first and absolute (unconditional) probability is an offshoot.
- He intended to develop an essentially *inductive* logic, since he was mainly interested in probability as a tool for making inferences rather than for developing scientific models [39].

In spite of the ample development of his theory, as expressed in its main reference [4], his overall programme is often considered not fully successful and his approach has been progressively abandoned and confused in the general Bayesian framework [27].

Furthermore, interestingly enough, the posthumous (gigantic) book that collects most of the work of Jaynes [31], an eminent exponent of (neo)Bayesian School, was titled "Probability: the logic of science", thus recalling the ambitious programme of Carnap. Yet he did not provided, as Carnap instead did, a formal treatment for this interpretation, that rather expressed a general flavour, an attitude.

So, with these – though shortly summarised – premises, does it still make sense to purse again a *logicistic* approach to probability?

We think it does, since our starting point differs substantially from Carnap's, in that we take a different position in regards of the two above mentioned starting points. That is to say that

- We regard probability as an inherently mono-argumental function, that is we start from absolute probability, and
- We try and develop a logic *tout court*, rather than an inductive one, at least for statement expressed through a first-order language.

Lastly, we also try and show how these ideas can be fruitfully applied in measurement science.

3. The proposed framework

3.1. Propositional and predicate logic

Logics studies the laws of reasoning[4] [9,20,35]. In the present day different types of logic are considered, depending upon the aspect of reasoning that is of major interest

[2] This stems from two main assumptions: that subjective assumptions may rely on some kind of inter-subjective agreement, and that the weight of objective data progressively prevails over that of subjective assumptions, as long as the amount of available data increases.

[3] The term "orthodox" statistics to denote the approach that refers mainly to Fisher [5] and his school was proposed by Jaynes [31]: we prefer it to the term "classic", also used with the same meaning, since we prefer to reserve this latter term to authors such as Gauss and Laplace [39].

[4] We present here a very short and totally informal introduction to logics, since we feel that it may perhaps be useful to some readers for understanding what follows in an easier way. We apologise with experts in logic for possible imprecision and (sure) superficiality. Readers already familiar with this subject may move directly to Section 3.2 or even to Section 3.3.

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