Measurement 79 (2016) 276–284

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Dynamics and stability – A proposal for related terms in Metrology from a mathematical point of view

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A R T I C L E   I N F O

Article history:
Available online 22 July 2015

Keywords:
Dynamic system
Time dependent signal
Property
Behaviour
Stable
Stationary

A B S T R A C T

We describe measurement quantities and measurement processes verbally by using everyday terms, often sanctioned by international standards and guides. However, we apply some of these terms inconsistently as compared to definitions in base sciences like Logic and Mathematics, Signal and System Theory, Stochastics and Statistics, Estimation and Optimisation Theory. This is especially true for a group of terms in Metrology, namely for the terms *kinetic, static, time dependent, time invariant, stable, stationary, drifting* etc., which all populate the general environment of the main term *dynamics*. The paper explores systematic relations between these and similar terms and discusses the aptitude of their implementation in everyday practice. Cardinal point is the term *dynamic*, which will be investigated thoroughly. So, derived terms can be defined, incorporated and judged accordingly. For the field of Metrology some new and, maybe, surprising results arise, worth being considered in the near future.

1. Introduction

Measurement instrumentation, measurement procedures and measurement results fulfil well-defined requirements. We certify them by qualitative verbal descriptions and/or by quantitative logical and mathematical relationships, according to accepted rules. Both, qualitative and quantitative results must be equally trustworthy.

Here, the focus lies on verbal communication in Metrology, which should not just rely upon any commonly accepted habit. For example, we state colloquially that certain situations and procedures are *stable* or *unstable*. Sometimes data are reported as *drifting*. Moreover, many instances appear *nonstationary*. Are such and other descriptions identical, equivalent, synonymous, similar or related? Do they mean *dynamic*? Finally, what is the logical and/or mathematical base? Of course, these terms may serve as hints for the time being. However, what shall we do, once we have to deliver robust information? We ask for specific meaning behind these terms, regularly used in everyday metrological practice. It is obvious that such an endeavour will narrow the meaning of the everyday terms concerned and will ban some of them from the scene.

The subject area of dynamic observation, measurement and analysis is steadily increasing. This is especially true for the fields *Dynamic Identification* (Calibration), *Dynamic Measurement* and *Dynamic Reconstruction*. They all have an impact on measurement errors and measurement uncertainties. Yet, the official terminology in Metrology concerning these items stays behind on a rather low level.

We would appreciate a concept, based on systematic and coherent tools, which would be applicable to all fields of science and would thus foster top-down and holistic considerations. Indeed, there are facilities in base sciences, like Logic and Mathematics, Model Development and Identification (Calibration), Signal and System Theory,

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http://dx.doi.org/10.1016/j.measurement.2015.07.026
0263-2241/

Stochastics and Statistics, Estimation and Optimisation [1–5,14]. However, one has to compile knowledge and skills, which are dispersed in these fundamental fields, in order to make them persuasive and thus applicable to metrological needs.

Interestingly enough the basic Guides of Metrology like the GUM [6] and the VIM [7] deliver no, or just a few helpful definitions concerning the field of consideration. Even the Standards of Statistics refrain from offering significant and precise support, when dealing with these terms [8,13].

The following sections assemble, classify and rigorously define meaningful protagonists around the term “dynamic”. This is done in a structured way in order to disclose their interrelations and dependencies.

Section 2: *Signals and Systems*. We know that some of the terms in question refer to *signals* and some of them to *systems*. Therefore, we highlight the difference between the interesting real world *process* and its model, the abstract *system*, on the one hand and the difference between the interesting real world *quantity* and its model, the abstract *signal*, on the other hand.

If we are able to define and to empirically determine *properties* and *behaviour* of those abstract signals and systems, we ascribe them to the real world quantities and processes.

We consider measurement errors and measurement uncertainties to be signals too, although they have no counterpart in the real world: Nature does not know errors and uncertainties. They are defined, abstract quantities, like efficiency, comfort, sustainability and so on.

Section 3: *Description of the Dynamic System*. Naturally, the term *dynamic* is the pivot for all phenomena that involve time dependencies. What does *dynamic* actually mean? Which other terms depend on the adjective *dynamic*? When is the term *dynamic* applied wrongly? In order to clarify the situation, we will present a concise survey of this topic, based on logical and mathematical concepts: Some new aspects will appear, so that we will be able to define and classify other terms from scratch later on.

The resulting definitions are easily transferrable from the time domain to the space domain. For clarity reasons, this is not done here however.

Section 4: *Properties and Behaviour of the Dynamic System*. The important differences between the properties of a system and the behaviour of a system are highlighted. They are intuitively accepted but not properly applied in everyday practice. A signal, coming from a system, has properties too, but does it have a behaviour of its own?

Section 5: *Time Dependent Terms*. The subsequent section will concentrate on three unpopular terms in Metrology, namely *instability*, *nonstationarity* and *drift*. Here, probability concepts of Stochastics and Statistics come directly into action. Again, the question is, whether signals and/or systems are involved at least.

Summary, References and some Terms and Symbols complete the text.

Remark: Only a few remarks on common terms in everyday practice will show the variety of meaning in the diverse fields of Metrology. A complete listing would be impossible. The emphasis of this survey is placed on mathematical substantiation.

2. Signals and systems

Terms like stability, stationarity and drift are used in practice to describe properties of *signals* and of *systems* as well. However, *properties* of systems primarily show up in their *behaviour*, and their output signals are only consequences. So, one has to distinguish carefully between definitions of signals and definitions of systems.

2.1. Signals are models of quantities

A signal (state, parameter, trajectory, transition, transient, propagation, walk, motion, movement, change, evolution, solution, evolvement, development, outcome, path, course, phase, orbit, observation, history, phenomenon, data, effect, event, instant, wave, pattern, picture) *describes* verbally and/or mathematically a *natural* or *man-made* and *man-defined quantity*.

2.2. Systems are models of processes

At first sight, a system seems to describe a natural or man-made real world process; however, this is only indirectly the case. In fact, *mathematical models of processes*, which are denoted as *systems*, describe *abstract relations* and *correlations* between defined *state* and *inner quantities* of a process and defined *external quantities* according to the cause and effect principle. By external quantities, we mean *input and output signals*. Systems do not describe any physical artefact or functional interconnection: In this concept, *systems* (*models*) *are hardly process oriented*, *but rather quantity oriented* [9].

The well-known example of a *dynamic system of second order* may illustrate this seemingly uncommon perception: A mass-damper-spring process and an inductance–capacitance–resistance process share exactly the same mathematical model. Only the *Physical Units* of the particular signals and parameters, which normally do not show up in the model, move it in the neighbourhood of a dedicated physical process after all, and make it identifiable.

As a consequence, many of so-called *soft sciences*, which do not know physical artefacts, get access to the art of quantitative Model Development, because we define *systems as relations* and *correlations* between quantities only.

So, the basic concept of relations between signals concerning a dynamic system admits all types of signals, which means, models of real physical quantities, defined virtual signals without real counterparts, and errors and uncertainties as well.

Signal and System Theory contains tools that covers both, models of quantities (signals) and models of processes (systems). *State Space Description* elegantly describes a large variety of processes within the Signal and System Theory. In addition, *Linear Algebra* provides algorithms to reveal the behaviour of dynamic systems quantitatively by *Simulation*.

3. Description of the dynamic system

Concerning dynamic systems, many verbal and/or mathematical depictions are only approximately correct,