Measurement 79 (2016) 285–301

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

# Measuring risk-aversion: The challenge

P.J. Thomas *

*Safety Systems Research Centre, Queen's School of Engineering, University of Bristol, Queen's Building, University Walk, Clifton BS8 1TR, United Kingdom*

ARTICLE INFO

*Article history:*
Received 21 October 2014
Received in revised form 17 July 2015
Accepted 27 July 2015
Available online 7 August 2015

*Keywords:*
Risk-aversion
Utility
Soft measurement
Economic measurement
Risk
Decision science

ABSTRACT

Risk-aversion is advanced as a measure of the feeling guiding the person who faces a decision with uncertain outcomes, whether about money or status or happiness or anything else of importance. The concepts of utility and, implicitly, risk-aversion were used first nearly 300 years ago, but risk-aversion was identified as a key dimensionless variable for explaining monetary decisions only in 1964. A single class of utility function with risk-aversion as sole parameter emerges when risk-aversion is regarded as a function of the present wealth, rather than subject to alteration through imagining possible future wealths. The adoption of a single class allows a more direct analysis of decisions, revealing shortcomings in the use of conventional, Taylor series expansions for inferring risk-aversion, over and above the obvious restrictions on perturbation size. Dimensional analysis shows that risk-aversion is a function of three dimensionless variables particular to the decision and a set of dimensionless character traits, identified later as the limiting reluctance to invest and the lower threshold on risk-aversion. The theoretical framework presented allows measurement of risk-aversion, paving the way for direct, evidence-based utility calculations.



## 1. Introduction

Risk-aversion is a fundamental parameter determining how much satisfaction or utility we obtain from an experience, from a good or from money. It establishes the shape of the utility function that quantifies how much satisfaction or utility we derive. Before attempting to measure it, it is obviously necessary to know what risk-aversion is and have a proper model of how it acts through utility to affect people's decisions. The possibility then arises of using the decision as the measured parameter and inferring the risk-aversion that must have been in place for that decision to have been made.

Unfortunately there has been incomplete agreement in the past on the behaviour of risk-aversion, even on whether and when risk-aversion can be regarded as constant. This lack of clarity has led to a number of different classes of functions being accorded the status of utility function, and this uncertainty has affected existing measurement methods, as will be explained. Thus the first challenge of this paper is to establish a realistic understanding of risk-aversion and its behaviour as a prelude to its accurate measurement.

The subject will be introduced by exploring the ways that the ideas of utility and risk-aversion have developed. The concepts are typically applied to the two canonical cases: the purchase of insurance and the purchase of a lottery ticket, which stand as proxies for additional types of decision under uncertainty. The conventional way of estimating the individual's risk-aversion in these cases uses a Taylor series expansion about the utility of the individual's starting wealth, as will be shown in Section 3. The required assumption of small deviations from that wealth brings with it significant limitations, but has the advantage that the form of the utility function does not need to be specified.

* Tel.: +44 117 331 5830; fax: +44 117 954 5208.
*E-mail addresses:* pjt3.michaelmas@gmail.com, philip.thomas@bristol.ac.uk

http://dx.doi.org/10.1016/j.measurement.2015.07.056
0263-2241/

A secondary advantage accrues in that a Taylor series expansion makes it unnecessary to consider whether or not the individual alters his risk-aversion when he is making a pairwise comparison between the utility of two possible outcomes. While there are good reasons for considering the risk-aversion of a wealthy person generally to be different from the risk-aversion of a poor person, it is argued in [1] that it is not feasible for a rich person considering the effect of a large insurance loss to experience the risk-aversion felt by a person subsisting at the post-loss level of wealth already. If it is accepted that risk-aversion is a feeling that develops from the experience of living in a given condition of life, lack of the required detailed knowledge and feel would prevent a rich person considering insurance thinking himself fully into the role of a poor person when evaluating his wealth after a substantial loss, even if he were minded to do so and however much he appealed to his imagination. Exactly the same argument applies to the poor man considering a lottery, and to people of all gradations of wealth in between. Nor is it sufficient to have lived in a different state of wealth in the past, since the feelings that the previous condition produced will be remembered so imperfectly that it would not be possible to develop the corresponding level of risk-aversion, even if the person wished so to do. A more realistic model is adopted in this paper. It is expected that the decision maker will vary his risk-aversion during the course of his pondering on his decision, but that risk-aversion will stay constant during each pairwise comparison of the outturn utilities resulting from the adoption or non-adoption of a particular course of action. This model is of strong economic importance, since it will be shown that the associated utility functions must then be of one class only, namely the Power utility, with risk-aversion as sole parameter.

Dimensional analysis will be used to clarify what can and should be measured in the insurance and lottery cases. Two additional, dimensionless parameters are recommended for measurement, which, like risk-aversion, are particular to the person. They are both limiting values: the first being the individual's limiting reluctance to invest (a scaled version of before and after utility differences) while the second is the individual's lower threshold on risk-aversion.

Worked examples will be given of possible measurement scenarios, both for lotteries and for insurance. Problems with the existing methods based on Taylor series expansions will be highlighted, including the rather striking fact that they fail to measure the right parameter in the case of insurance. Guidance will be given on how to obtain a good signal to noise ratio when measuring risk-aversion. Finally, the residual difficulties will be brought out of measuring a parameter that is personal to the individual, and that will vary according to the importance of the decision.

## 2. Development of the concepts of utility and risk-aversion

The study of utility as a way of explaining people's actions has a long and illustrious history, having gained the attention of a series of distinguished scholars, from Daniel Bernoulli to John von Neumann. The derivatives of utility have a particular importance in economic theory. The first derivative will be discussed now and the second derivative later in this Section. The first derivative, known to economists as 'marginal utility', is defined by the Encyclopaedia Britannica [2] as "the additional satisfaction or benefit (utility) that a consumer derives from buying an additional unit of a commodity or service". The utility to a consumer of an additional unit of a product is normally taken to be inversely related to the number of units of that product he already owns.

The concept of marginal utility was key to Jevons's solution [3] of the 'paradox of value', which had perplexed economists until the late 19th century and is illustrated in the much higher monetary value attached to diamonds as compared with the same mass of bread, even though the latter is an important dietary component and the former merely an adornment. How can this be? Marginal utility allowed the following explanation. People are attracted to diamonds but the fact that they are scarce means that only a small number of people can have many of them. Under these conditions the marginal utility of diamonds to those people with few of them will be high, which explains why they command a high price. On the other hand, bread is in plentiful supply, so that customers for bread can soon possess enough to satisfy their most pressing need. As a person's appetite for bread becomes satisfied, so the additional utility of a further slice, the marginal utility, will go down, with the result the price he will be prepared to pay will fall. A glut of bread could drive its price down to practically zero, since all or almost all potential customers would have enough bread already [2,4].

Despite the success of marginal utility in providing a conceptual framework for understanding the paradox of value, there was clear difficulty in measuring quantitatively the utility that a person received from consuming a product. So shortly after Jevons's work, Edgeworth [5] began the development of indifference theory. Whereas utility theory assumes, at least in principle, the numerical measurability of the difference in the utility conferred by two options on a person or an organisation, indifference theory rests on the weaker assumption that the person can specify only which option yields him the higher utility. Further important work, carried out by Pareto in the 1900s [6] and, thirty years later, by Hicks [7] led to the wide acceptance of indifference theory.

But shortly after Hicks' *magnum opus* was published, the possibility of numerical measurement of utility was revived by von Neumann and Morgenstern in their highly influential book, *Theory of Games and Economic Behaviour* [8], published in 1944. In a direct challenge to Hicks's indifference curve methods, they claimed that:

> "the treatment by indifference curves implies either too much or too little: if the preferences of the individual are not all comparable, then the indifference curves do not exist. If the individual's preferences are all comparable, then we can even obtain a (uniquely defined) numerical utility, which renders the indifference curves superfluous.".