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Commparison Between Different Methods of Control of Ball and Plate System with 6DOF Stewart Platform

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Abstract: In this paper, we tackle the control problem of ball and plate system (BPS) with 6 DOF Stewart platform. The BPS is a typical multi-variable nonlinear system which is a two dimensional expansion of the ball and beam system. Four strategies are proposed for static and dynamic position tracking: PID, LQR, Sliding Mode and Fuzzy controller. The results of simulation and also the validation on real system are provided. The comparison between the proposed strategies is presented based on the performance of the tracking.

Keywords: Nonlinear control, Ball and plate system; Stewart platform, Sliding mode control, Fuzzy control, LQR.

1. INTRODUCTION

The ball and beam system is a well-known problem for nonlinear control where the system is under-actuated and has two degrees of freedom, while the ball and plate system (BPS) can be considered as an extension of that system consisting of a ball that can roll freely on a plate. Therefore, the BPS has four degrees of freedom and it is more complicated due to the coupling between the variables. The complexity of the problem increases when the system is held on 6 DOF Stewart platform. In our case, the experimental system includes a plate fixed on the moving Stewart platform, a ball, six motors and their driving system. A touch pad is used to measure the position of the ball. This system can be considered as a big challenge to test various nonlinear strategies.

However, most of the previous works focused on the two dimensional Electro-mechanical ball and plate system (all the references). In Knuplez (2003), a controller design based on classical and modern control theory was proposed. A supervisory fuzzy controller of two layers was proposed in Ming (2006) to study the motion control in static and dynamic tracking. A state observer was used in Hongrui (2008) to estimate ball velocities while the position of the ball was regulated with double feedback loops in Wang (2008). The controllability on Poisson manifolds of the system was studied in Siyan (2009). The disturbance rejection topic was also tackled by some works. In Huida (2009), an active control is applied to the trajectory tracking and in Xiucheng (2009) PID neural network controller based on genetic algorithm was proposed. In Hai-Qi Lin (2014), a controller was designed to ensure the stability employing a loop shaping method based on Normalised Coprime Factors perturbation model. Besides, some works employed the sliding mode control (Dejun (2008), Hong Wei (2012)). In Ghiasi (2012), an optimal robust controller was designed for the trajectory tracking and only the results of simulation were presented. In this paper, we study the ball and plate system where the plate is mounted on Stewart Platform with 6 DOF. Four controllers designed and evaluated in static and dynamic tracking: Classical controller PID, Linear quadratic regulator (LQR), Sliding Mode Controller and Fuzzy Controller. The inverse kinematic of the Stewart platform was used to calculate the instantaneous inclination of the plate. The rest of the paper is organized as follows. Section 2 introduces the modeling of the ball and plate system. Section 3 discusses the design of the four methods of control and presents simulation. Finally, the validation on real system is presented in Section 4.

2. MATHEMATICAL MODELING

The following mathematical equations are based on (X.Fan (2004), K. Kyu Lee (2008)). The Euler-Lagrange equation of ball-plate system can be written as following:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \tag{1}$$

,where q_i stands for i-direction coordinate, T is the kinetic energy of the system, V is potential energy of system and Q is composite force.

The system has four degrees of freedom; two derived from the motion of the ball and two for the inclination of the plate. Here we assume that the generalized coordinates of the system are x_b and y_b (the position of the ball on the plate) and α and β (the rotations of the plate). The kinetic energy of the ball consists of its both rotational with respect to its center of mass and translational energy:

$$T_b = \frac{1}{2} m_b \left(\dot{x}_b^2 + \dot{y}_b^2\right) + \frac{1}{2} I_b \left(\omega_x^2 + \omega_y^2\right) \tag{2}$$

Where m_b is the mass of the ball and I_b is the moment of inertia of the ball.

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As the ball is assumed to rotate without slippage, therefore:

$$\dot{x}_b = r_b \omega_y \quad , \quad \dot{y}_b = r_b \omega_x \tag{3}$$

where r_b denotes the radius of the ball. By substituting equations (3) into equation (2) we will have:

$$T_b = \frac{1}{2}\left(m_b + \frac{I_b}{r_b^2}\right)\left(\dot{x}_b^2 + \dot{y}_b^2\right) \tag{4}$$

The kinetic energy of the plate (assuming that there is no spin around z axis)includes its rotational kinetic energy and the rotational kinetic energy of the ball and it can be written as:

$$T_p = \frac{1}{2}\left(I_p + I_b\right)\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + \frac{1}{2}m_b\left(x_b\dot{\alpha} + y_b\dot{\beta}\right)^2 \tag{5}$$

The relative potential energy of the ball to horizontal plane passing by the center of the inclined plate is:

$$V_x = m_b g x_b \sin\alpha \quad , \quad V_y = m_b g y_b \sin\beta$$

Where g is the gravity acceleration. By applying the Euler-Lagrange's equation, we obtain the mathematical model for the ball and plate system as follows:

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{x}_b - m_b\left(x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}\right) + m_b g \sin\alpha = 0 \tag{6}$$

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{y}_b - m_b\left(y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta}\right) + m_b g \sin\beta = 0 \tag{7}$$

$$\begin{aligned}\tau_x = \left(I_p + I_b + m_b x_b^2\right)\ddot{\alpha} + 2m_b x_b \dot{x}_b \dot{\alpha} + m_b x_b y_b \ddot{\beta} \\ + m_b \dot{x}_b y_b \dot{\beta} + m_b x_b \dot{y}_b \dot{\beta} + m_b g x_b \cos\alpha\end{aligned} \tag{8}$$

$$\begin{aligned}\tau_y = \left(I_p + I_b + m_b y_b^2\right)\ddot{\beta} + 2m_b y_b \dot{y}_b \dot{\beta} + m_b x_b y_b \ddot{\alpha} \\ + m_b \dot{x}_b y_b \dot{\alpha} + m_b x_b \dot{y}_b \dot{\alpha} + m_b g y_b \cos\beta\end{aligned} \tag{9}$$

Equations (6) and (7) describe the movement of the ball on the plate and they reflect the relationship between the acceleration of the ball and the rotational angle and the angular velocity of the plate and equations (8) and (9) show the effect of external torque on the whole system.
It is difficult to control such a highly nonlinear system. Besides that, in our system, the considered inputs are the angles α and β. Therefore, for simplification we can focus only on the equations (6) and (7).
In the steady state, the plate should be in the horizontal position, and the two angles α and β must equal to zero. We derive the linearised model of system in a neighbourhood of this working state assuming that the range of the rotational angles of the plate is $[-5°, +5°]$. Therefore, we can use the approximation of the sine function, i.e. $\sin\alpha \approx \alpha$,$\sin\beta \approx \beta$ and Equations (6) and (7) become:

$$x : (m_b + \frac{I_b}{r_b^2})\ddot{x} + m_b g \alpha = 0 \tag{10}$$

$$y : (m_b + \frac{I_b}{r_b^2})\ddot{y} + m_b g \beta = 0 \tag{11}$$

Where $I_b = \frac{2}{5}m_b r_b^2$.
If we consider $X = (x, \dot{x}, y, \dot{y})$ as a state vector of the system and the angles of the plate as the control input, we obtain the following state space representation:

$$\dot{X} = AX + Bu$$
$$Y = CX$$

where:

$$A = \begin{pmatrix} 0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0 \end{pmatrix}, B = -\frac{5}{7}g\begin{pmatrix} 0&0\\1&0\\0&0\\0&1 \end{pmatrix}, C = \begin{pmatrix} 1&0&0&0\\0&0&1&0 \end{pmatrix}$$

The above system is completely controllable and observable because the ranks of the controllability matrix and of the observability matrix are both 4 which equal to the dimensions of the considered system state. We consider this representation in the design of the controllers in the next section.

3. THE DESIGN OF THE CONTROLLERS

We proposed four methods to control the static and dynamic position of the ball on the plate. We fixed some requirements in order to compare between the performances of these methods. We consider the response time t_s, the maximum overshoot D_{100} and the steady state error e_{ss}. The limits are given by:

$$\begin{cases} t_s & \leq 3sec \\ D_{100} & < 5\% \\ e_{ss} & \leq 2mm \end{cases}$$

The following simulation results are tested in MATLAB 2009 program.

3.1 PID Controller

In order to stabilize the ball in a desired position with respect to the requirements, we designed PID controller for x-axis and another one for y-axis as they are expressed in two separate differential equations. Fig.1 shows the simulation result for step response with the obtained controller for the position of the ball on the x-axis. We got: $t_s = 2.5sec$, $D_{100} < 5\%$ and $e_{ss} = 0$, with : $K_p = -0.3$, $K_d = -0.337$.

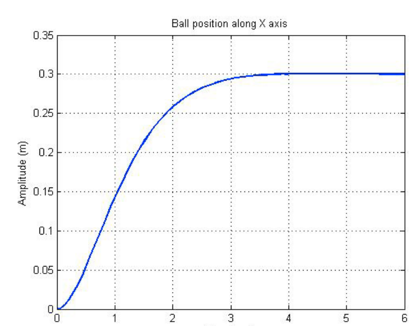

Fig. 1. Position of the ball on the x-axis (PID).

In Fig.2, the simulation result for the tracking of a desired circle with radius of 5 cm is presented where we obtained an error less than 3 mm using these parameters ($K_p = -0.52$, $K_d = -0.35$)

3.2 LQR Controller

The second control method is a Linear Quadratic controller. Here, the feedback gain is based upon the minimization of a quadratic cost function:

$$J = \frac{1}{2}\int_0^\infty \left(x^T Q x + u^T R u\right) dt$$