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Dynamical modeling for electric hot water tanks

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Abstract: To quantify the potential of electric hot water tanks (EHWT) in general demand response programs, there is a need for models with prediction capabilities at a reasonable computational cost. As can be experimentally observed, the input-output response of EHWT is relatively complex. This paper presents two models of EHWT, one in the form of two simple one-dimensional partial differential equations and the other as a hybrid system decoupling the phenomena acting on the EHWT, in sequences. An experimental validation compares the performance of these models. The conclusion is that the hybrid model is more accurate and less computationally intensive.

Keywords: Electric hot water tank, distributed parameter systems, hybrid modeling, nonlinear systems, load shifting.

1. INTRODUCTION

The increasing share of intermittent renewable electricity sources in the energy mix (Commission [2011], Edenhofer et al. [2011]) reveals troublesomefor managing the electricity production-consumption equilibrium. This status can be observed at national and local levels. Demand Side Management (DSM), which is a portfolio of techniques aiming at tailoring consumers' demand, is a promising solution for such concerns (Palensky and Dietrich [2011]). A key factor in developing DSM is the availability of energy storage capacities. For this reason, network operators and electricity producers are developing new ways of storing energy. In this context, the large pools of electric hot water tanks (EHWT) found in numerous countries (the market share of electric heater being 35% in Canada, see Aguilar et al. [2005], 38% in the U.S, see Ryan et al. [2010], and 45% in France, see MSI [2013]) is particularly appealing. For load shifting applications, its large storage capacities, the flexibility yielded by its geographically scattered characteristic and its functioning are key enabling factors.

EHWTs heat water over relatively long periods of time. To minimize cost, electricity is used in the night time (one period when electricity price is low), while hot water is used in the next day-time. More advanced timing strategies are believed to generate further cost-reductions. Design of these strategies requires dynamical models, e.g. to determine optimal heating periods in response to fluctuating prices of electricity. In this paper, we develop one such model.

An EHWT can be seen as a two inputs, single output dynamical system. The two inputs are the heating power and water outflow (or drain). The output is the distribution of temperature of water in the tank, which can be used to define, for optimization purposes, quality of service indexes. This simple description should not suggest that the internal system dynamics are straightforward. In this paper, we will consider a natural performance index, the "available energy", representing the total energy contained in the water whose temperature is above some prescribed threshold (e.g. 40^{o}C). The rationale behind this choice is that hot water can be used by blending it with cold water for all domestic purposes, provided its temperature is above the desired threshold. As will appear, accurate forecasting of this variable requires advanced modeling.

In the literature (Blandin [2010], Kleinbach et al. [1993], Zurigat et al. [1991]), hot water storages are modeled as vertical cylindrical columns driven by thermo-hydraulic phenomena: heat diffusion, buoyancy effects and induced convection and mixing, forced convection induced by draining and associated mixing, and heat loss at the walls. Most existing models are either *i)* one-dimensional superposition of layers (see e.g. Blandin [2010]), or *ii)* three or two-dimensional (using rotational symmetry) models, often using a discretization for numerical simulation purposes such as computational fluid dynamics (Blandin [2010], Han et al. [2009], Johannes et al. [2005]) or *iii)* so-called zonal models often based on the software TRNSYS (Johannes et al. [2005], Klein et al. [2010]). These models, although accurate, are numerically intensive and do not fit with our requirements of numerical efficiency. On the other hand, when overly simplified, these layers models fail to reproduce some physical phenomena whose effects are observed in practice. This is particularly true in

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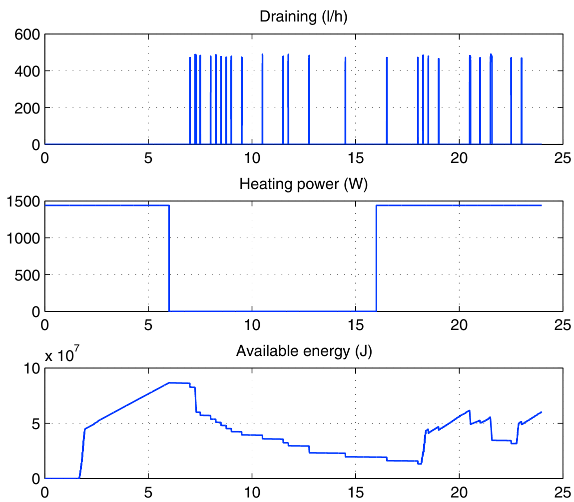

Fig. 1. Available hot water as a function of heating and draining on a 24h run (experimental data). The system is initially uniformly cold.

our context when one wishes to introduce heating in the dynamics.

A complexity trade-off must be found. Interestingly, a careful study of the physical principles at stake in the system suggests some simplifications. The buoyancy effects lead to the so-called stratification phenomenon (Han et al. [2009]), causing horizontal homogeneity of the temperature profile, increasing with height. This effect is dominant and allows one to consider only one-dimensional models. Following this approach, the first model we briefly develop here extends an existing one-dimensional convection-diffusion linear equation modeling the draining convection and its mixing developed in the 1980s (Zurigat et al. [1988, 1991]). In details, to the classic governing equation, we add a nonlinear velocity term given by empirical laws lumping various phenomena (turbulent natural convection due to heating in particular). Further, we introduce heating power as a source term. Experimental data illustrate the relevance of this modeling. The model is concise, and relatively accurate. However, several improvements are possible.

Then, in a second step, we decouple heating and draining effects and develop a model based on the decomposition of the dynamics according to the dominant effect at stake. We distinguish three phases: heating, draining and rest. For heating, we reproduce a behavior observed in experimental data in which the temperature increases first at the bottom of the tank forming a spatially uniform temperature distribution which gradually extends itself upwards to the top of the tank. We explain this observation by a model of buoyancy-induced forces, generating a local natural convection phenomenon. This homogeneous zone is followed by an increasing profile of temperature in the upper part of the tank, remaining untouched due to stratification (heat diffusion being neglected in this case). Draining is treated as a convection parameter and its associated mixing effects are reproduced by a diffusion term following the approach of Zurigat et al. [1991]. We model the effects

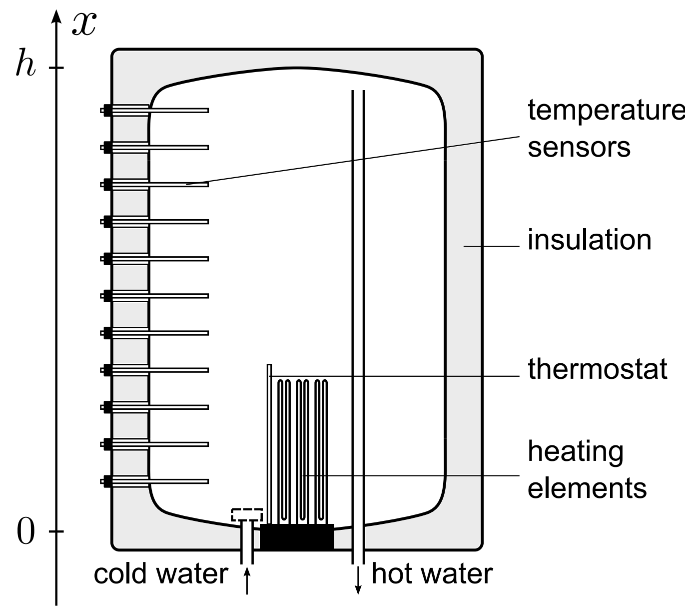

Fig. 2. Schematic view of an EHWT.

of the water nozzle which creates a mixing zone of varying temperature and volume. The cascade represents a Stefan problem (Fasano and Primicerio [1977a,b]). Finally, rest phases are simply driven by diffusion and loss. Sequencing the three phases constitutes a hybrid model.

This hybrid model is the main contribution of this article. It kindly reproduces experimental data presented in this article, and permits to compute the dynamics of the "available energy", defined earlier, in response to the water drain and the power injected in the tank. A typical scenario is reported in Fig.1. As it is visible, the input-output behavior of the system in somewhat complex although not counter-intuitive[1] .

The paper is organized as follows. After having described the first model, we illustrate it by means of simulations and compare it against experimental data in Section 2. In this study, a typical 200L tank (equipped with spatially distributed internal sensors) with realistic scenarios of draining and heating is employed. Section 3 is dedicated to the presentation of the hybrid model which is the main contribution of the paper. Comparative studies reported in Section 4 conclude that this hybrid model is more accurate and more computationally efficient.

2. PRELIMINARY CONSIDERATIONS AND FIRST MODELING

2.1 General considerations on water tanks and stratification

A typical EHWT is a vertical cylindrical tank filled with water in which a heating element is plunged at the bottom end (see Fig. 2). The heating element is pole-shaped, and its length is relatively large, up to one third of the tank. Cold water is injected at the bottom while hot water

[1] Heating water takes time. Starting from a uniformly cold tank, the output of the system (the "available energy") remains identically equal to zero for hours. Then it jumps, and steadily increases. Draining causes steps down on the output, and also causes some internal mixing which is non negligible.