Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-11 (2015) 120–125

A Generalized Piecewise Quadratic Lyapunov Function Approach to Estimating the Domain of Attraction of a Saturated System ⋆

Yuanlong Li * **Zongli Lin** **

* *Department of Automation, Shanghai Jiao Tong University, and Key Laboratory of System Control and Information Processing of Ministry of Education, Shanghai 200240, China (e-mail: liyuanlong0301@163.com)*

** *Charles L. Brown Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA 22904-4743, USA (e-mail: zl5y@virginia.edu)*

Abstract: This paper revisits the problem of estimating the domain of attraction of a saturated system with an algebraic loop. A piecewise quadratic Lyapunov function of an augmented state vector composing of the system state and the saturated input has been proposed for use in estimating the domain of attraction for a saturated system. Considering the relationship between the system states and the saturation function, we propose in this paper a generalized piecewise quadratic Lyapunov function, which results from adding a term that characterizes the regional sector condition of the saturated input to the piecewise quadratic Lypunov function. The matrix associated with the generalized piecewise quadratic Lyapunov function is not required to be positive definite, and thus a set of less conservative stability conditions are established, from which a larger estimate of the domain of attraction can be obtained. Simulation results indicate that the effectiveness and superiority of the proposed method.

© 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Domain of attraction, input saturation, generalized piecewise quadratic Lyapunov functions

1. INTRODUCTION

For a linear system under saturated linear feedback, whose open-loop poles are not all in the closed left-half plane, the analytic characterization of its domain of attraction, in general, seems impossible. An alternative approach is to estimate the domain of attraction with a contractively invariant set, which is a subset of the domain of attraction [1, 2, 3, 5, 7, 8, 9, 11, 12]. One such contractively invariant set is the polyhedral invariant set, which could result in a good approximation of the domain of attraction. For example, a polyhedral SNS invariant set, which embeds the characteristics of saturated linear feedback, was proposed in [1] as an estimate of the domain of attraction of a discrete-time saturated linear system.

Another commonly used invariant set is the ellipsoidal or ellipsoidal-like invariant set. Such an invariant set can be described as a level set of a quadratic or non-quadratic Lyapunov function. As the most commonly used Lyapunov function, the quadratic Lyapunov function induces ellipsoidal invariant sets which have been widely used as estimates of the domain of attraction of linear systems with saturated linear feedback [5, 6, 7, 9]. Many attempts have been made to reduce the conservatism associated with quadratic Lyapunov functions, and some non-quadratic Lyapunov functions have been well investigated for estimating the domain of attraction of saturated linear systems. For example, a composite quadratic Lyapunov function is composed from a group of quadratic functions in [8]. A level set of this composite Lyapunov function is the convex hull of the corresponding level sets of the individual quadratic Lyapunov functions. A saturation-dependent Lyapunov function was proposed in [2] that takes into account the severity of the actuator saturation.

The non-quadratic Lyapunov functions mentioned above involve the composition of some quadratic functions of the system state. Recently, a class of quadratic functions of the augmented state vector that contains the system state and saturation/deadzone function of the input have been employed to estimate the domain of attraction of saturated linear systems [3, 4, 10, 12]. An integral of the saturation/deadzone function of the state is added to a quadratic Lyapunov function to form a Lure-Postnikov type Lyapunov function [4, 10]. This Lyapunov function can be equivalently written as a quadratic function of the augmented state vector with a positive definite block diagonal matrix, and has been generalized in [3] to a piecewise quadratic Lyapunov function of the augmented state vector with a general positive definite matrix. Furthermore, by using this piecewise quadratic Lyapunov function and

⋆ This work was supported in part by the National Natural Science Foundation of China under Grant Nos. 61221003 and 61273105.

2405-8963 © 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.09.170

the partitioning of the virtual input space that enables some special properties of saturation functions to emerge, a larger estimate has been obtained in [12]. Because of the dependence on the state of the saturated input, the positive definiteness requirement of the matrix in the piecewise quadratic Lyapunov function is apparently conservative. With this observation in mind, in this paper, we add a term that characterizes the regional sector condition of saturation function of the input to the piecewise quadratic Lyapunov function, and construct a generalized piecewise quadratic Lyapunov function without the positive definiteness requirement of the associated matrix. A set of matrix inequality conditions with less conservativeness are established for guaranteeing the contractive invariance of a level set of this new Lyapunov function. Simulation results show that the estimate resulting from this new Lyapunov function is significantly larger than those in [3, 9, 12].

The remainder of this paper is organized as follows. In Section 2, we review the existing piecewise quadratic Lyapunov functions [3, 4, 10] and propose a generalized piecewise quadratic Lyapunov function which incorporates the regional sector condition. In Section 3, we establish stability conditions under which the level set of the generalized piecewise quadratic Lyapunov function is an estimate of the domain of attraction. An optimization problem is formulated to maximize such an estimate. Section 4 provides some simulation results to illustrate the effectiveness of the results in Section 3. Section 5 concludes the paper.

Notation. For a square matrix A, $\mathrm{He}(A) := A + A^{\mathrm{T}}$. For two integers l_1 and $l_2 \geq l_1$, $I[l_1, l_2]$ denotes the set of integers $\{l_1, l_1+1, \cdots, l_2\}$. For an integer m, let $\mathcal{K}$ be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^m elements in $\mathcal{K}$. Suppose that these elements of $\mathcal{K}$ are labeled as K_i, $i \in I[1, 2^m]$. Let $K_i^- = I - K_i$. Clearly, $K_i^- \in \mathcal{K}$. Let I_m denote the identity matrix of dimension m, and $0_{n\times m}$ the $n \times m$ zero matrix. For a matrix $P \in \mathbf{R}^n$, $P = P^{\mathrm{T}} > 0$, $\mathcal{E}(P) := \{x \in \mathbf{R}^n : \ x^{\mathrm{T}} P x \leq 1\}$.

2. PRELIMINARIES

2.1 System description

Consider a system with saturation in the following form:

$$\begin{cases} \dot{x} = Ax + B\mathrm{sat}(u), \\ u = Cx + D\mathrm{sat}(u), \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^m$ is the input, and $\mathrm{sat} : \mathbf{R}^m \to \mathbf{R}^m$ denotes the saturation function defined as

$$\mathrm{sat}(u) = [\mathrm{sat}(u_1), \mathrm{sat}(u_2), \cdots, \mathrm{sat}(u_m)]^{\mathrm{T}},$$
$$\mathrm{sat}(u_j) = \mathrm{sgn}(u_j)\min\{1, |u_j|\}.$$

Many linear systems with saturation or deadzone components, such as linear systems subject to actuator saturation and with anti-windup compensation, can be transformed into the form of system (1). By the relationship $\mathrm{dz}(u) = u - \mathrm{sat}(u)$, where $\mathrm{dz}(\cdot)$ denotes the deadzone function, system (1) can be equivalently converted into a system with deadzone nonlinearity, which is considered in [3, 9]. When $D = 0$, system (1) reduces to a linear system with saturated linear feedback

$$\dot{x} = Ax + B\mathrm{sat}(Cx). \tag{2}$$

When $D \neq 0$, system (1) contains an algebraic loop,

$$u = Cx + D\mathrm{sat}(u). \tag{3}$$

This algebraic loop is said to be well-posed if there exists a unique solution u for each x. A necessary and sufficient condition for the well-posedness is that the values of $\det(I_m + (I_m - D)^{-1} D K_i)$, $i \in I[1, 2^m]$, are all nonzero and have the same sign. One can easily verify this condition by Claim 2 in [9]. Throughout this paper, the well-posedness of the algebraic loop (3) is assumed.

2.2 Treatments of the saturation function

Next, we review some treatments of the saturation function as found in [3, 5], which will be used for the stability analysis for system (1) in Section 3.

Lemma 1. Given $v = [v_1 \ v_2 \cdots v_m]^{\mathrm{T}} \in \mathbf{R}^m$ such that $|v_j| \leq 1$, $\forall\, j \in I[1, m]$, the following inequality holds for any diagonal matrix $S \in \mathbf{R}^{m\times m}$ satisfying $S > 0$,

$$(u - \mathrm{sat}(u))^{\mathrm{T}} S\, (\mathrm{sat}(u) - v) \geq 0, \ \ \forall\, u \in \mathbf{R}^m.$$

Denote $\dot{u} = \frac{du}{dt}$ and $\phi(x) = \frac{d\mathrm{sat}(u)}{dt}$. It is clear that

$$\phi_j(x) = \begin{cases} 0, & \text{if } |u_j| > 1, \\ \dot{u}_j, & \text{if } |u_j| < 1, \end{cases}$$

where u_j denotes the j^{th} element of u.

The following lemma describes two sector-like conditions for the saturation function, where the first sector-like condition was derived in [3], and the second one can be easily verified.

Lemma 2. For every diagonal matrix $S \in \mathbf{R}^{m\times m}$, the following equalities hold almost everywhere,

$$\phi^{\mathrm{T}}(x) S\, (\dot{u} - \phi(x)) \equiv 0, \tag{4}$$
$$\phi^{\mathrm{T}}(x) S\, (u - \mathrm{sat}(u)) \equiv 0, \tag{5}$$

where $\dot{u} = CAx + CB\mathrm{sat}(u) + D\phi(x)$.

2.3 Lyapunov functions

In estimating the domain of attraction for system (1), different Lyapunov functions have been utilized whose level sets are used as estimates of the domain of attraction. One frequently used Lyapunov function is the quadratic Lyapunov function of the system state

$$V_Q = x^{\mathrm{T}} P_Q x, \tag{6}$$

where $P_Q \in \mathbf{R}^{n\times n}$ is positive definite. The LMI-based conditions under which the ellipsoid $\mathcal{E}(P_Q)$ is a contractively invariant set, and hence an estimate of the domain of attraction of system (1), were established in [9] by using the quadratic Lyapunov function (6) and the convex hull representation of saturated linear feedback. The authors of [4, 10] incorporated an integral of deadzone function into the quadratic Lyapunov function of the input (6), and a Lure-Postnikov type Lyapunov function is formed as follows

$$V_L(x) = x^{\mathrm{T}} P_Q x + \sum_{j=1}^{m} \int_0^{u_j} \mathrm{dz}(\sigma) w_j d\sigma, \tag{7}$$

where u_j is the jth element of u, and $w_j > 0, \forall j \in I[1, m]$. Furthermore, the authors of [3] presented a piecewise Lyapunov function,

Download English Version:

https://daneshyari.com/en/article/712451

Download Persian Version:

https://daneshyari.com/article/712451

Daneshyari.com