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Geometric path following control in a moving frame ⋆

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Abstract: The paper describes an approach to the development of the geometric path following control for a dynamical model of the rigid body with unidirectional thrust. The popular example of such system is the dynamical model of the quadcopter. Desired path of movement in the space is represented by an intersection of two implicit surfaces. In this paper we assume that the desired path is attached to a movable frame. This is a natural extension of the classical approaches for stationary frames. Path following control problem is posed as a problem of maintaining the holonomic relationships between the system outputs. Control is synthesized using the differential geometrical method through nonlinear transformation of initial dynamic model. The main results presented are the model of spatial motion and relevant nonlinear control algorithms.

Keywords: Moving path following, coordinates transformation, set stabilization, nonlinear systems, motion control

1. INTRODUCTION

The paper considers the development of path following control system for a rigid body, that is, the problem of providing a motion along a given spatial path. With the advent of unmanned vehicles the path following control problem became even more urgent, because path following is an UAV major operating mode.

Some of the most popular methods adopted from the missile guidance and typically include the line-of-sight (LOS) approach[Breivik (2010),Breivik and Fossen (2005),Lekkas and Fossen (2014),Fossen (2011)]. These methods are actively improved and widely used in marine and aerial guidance systems.

Another popular approach is consideration of a guidance as a tracking system controlled by a reference model is presented [Aguiar et al. (2005),Lee et al. (2010)]. The path is generally set by a time-dependent function, which leads to practical problems when the object motion is behind or ahead of the program due to parametric uncertainties or external disturbances. To solve this problem, the path should be parametrized by the length instead of time, and dynamics of this parameter should be introduced in the system model. In this guidance approach widely used the notion of a virtual vehicle which moves on a geometrically defined path in [Aguiar et al. (2005),Lapierre and Soetanto (2007),Lapierre et al. (2003)]. The virtual vehicle is coupled to the motion of the real one via a some abstract link. This method rather easily realizes the motion along polynomial curves, which provides better path planning and more accurate path following.

An alternative path following methodology using vector fields was presented in [Lawrence et al. (2008),Nelson et al. (2007)]. This method has global convergence. The main disadvantages of this approach is that there is no general procedure for how to construct the vector field for an arbitrary curve.

Original approach to the path following is based on stabilization of invariant manifolds in state space based on feedback linearization [Nielsen et al. (2009), Hladio et al. (2013)] or passive-based control [El-Hawwary and Maggiore (2011), El-Hawwary and Maggiore (2013)]. Simply speaking, a transformation generating an attractor in state space is selected for the initial system. In path following context, the attractor is a desired path set in output coordinates. Then the designer should only stabilize this solution, which is much less demanding than creating a tracking system as in the first approach. As a control

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object, an autonomous robot is a multichannel nonlinear dynamic system. Control system of a mobile robot should generate control actions providing preset motion of the center of mass in operating area. The one of the methods for synthesizing the control algorithms was proposed by I.V. Miroshnik. It is based on the second approach and implies nonlinear transformation of robot model to the task-oriented coordinate system, which makes it possible to reduce the complex multichannel control problem to several simple problems of compensation of linear and angular deviations and then to find adequate control laws using nonlinear stabilization [Fradkov et al. (1999), Kapitanyuk and Chepinsky (2013), Kapitanyuk et al. (2014)].

Differentially geometric methods of nonlinear control theory [Fradkov et al. (1999), Kapitanyuk and Chepinsky (2013), Miroshnik and Nikiforov (1996), Miroshnik and Lyamin (1994),Miroshnik and Sergeev (2001), Kapitanyuk et al. (2014)] are used in the analysis method for these systems and synthesis of control algorithms solving the path following problem as a stabilization problem with respect to implicit space curve (Fig. 1).

One of the most interesting path following task is the moving path following control [Oliveira et al. (2013), Regina and Zanzi (2011)]. In this case the desired path is attached to a movable frame. This is a natural extension of the classical approaches for stationary frames. An application example of this task is the following by UAV of a moving ground vehicles(Fig. 2) [Regina and Zanzi (2011)]. In this work we would like to demonstrate the method of solution the similar tasks using the geometric path following framework based on the stabilization of sets. This article deals with further development of task-oriented approach inspired by works[El-Hawwary and Maggiore (2011), Hua et al. (2013)]. This article focuses directly on the synthesis of controllers without restricting the path planning method.

2. DYNAMIC MODEL AND STATEMENT OF CONTROL PROBLEM

Consider a underactuated dynamic model with unidirectional thrust. It is the simplest model such object as the quadcopter. This model is a very interesting object to study due to nonlinear coupling between translational and rotational motions.

$$\ddot{x}(t) = g - \frac{f(t)}{m}\bar{n}(t), \tag{1}$$

$$\dot{R}(t) = S(\omega(t))R(t), \tag{2}$$

$$J\dot{\omega}(t) + \omega(t) \times J\omega(t) = M_c(t), \tag{3}$$

where $x \in \mathbb{R}^3$ is the Cartesian position vector of the center of mass C in the inertial reference frame XYZ, $g \in \mathbb{R}^3$ is the vector of gravity, $m \in \mathbb{R}$ is a total mass of the rigid body, $f \in \mathbb{R}$ is the magnitude of the control forces, $\bar{n}$ is the vector of the unidirectional thrust in the inertial frame, $R \in SO(3)$ is the rotation matrix from the body-fixed frame to the inertial frame, $\omega \in \mathbb{R}^3$ is the vector of angular velocities in the body-fixed frame, $M_c \in \mathbb{R}^3$ is the vector of the control moments in the body-fixed frame, $S(\omega) \in SO(3)$ is the skew symmetric matrix with structure:

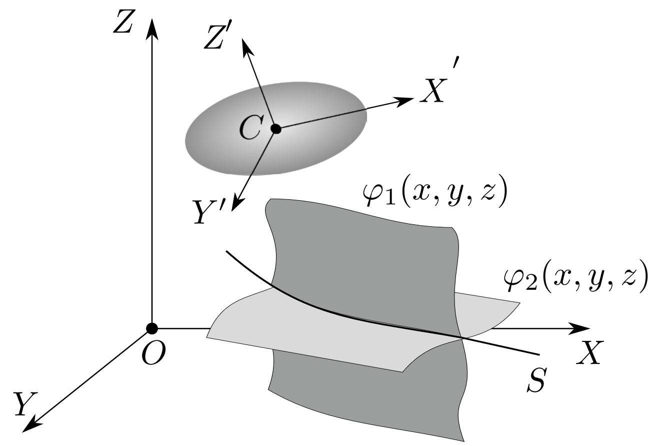

Fig. 1. Geometric path following task

$$S(\omega) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix}. \tag{4}$$

It should be noted that $S(a)b = a \times b$. For the quadcopter model the vector of thrust is defined as $\bar{n} = Re_3$, where $e_3 = [0,\ 0,\ 1]^\top$ is the unity vector along Z axis.

The desired path is a smooth segment of curve S (see Fig. 1) described as an intersection of two implicit surfaces

$$\varphi_1(x) = 0 \cap \varphi_2(x) = 0, \tag{5}$$

where φ_1 and φ_2 are smooth functions.

Tangential velocity along the desired curve is defined as

$$\dot{s} = (\nabla\varphi_1 \times \nabla\varphi_2)^\top \dot{x}, \tag{6}$$

where $\times$ is the vector product, $\nabla f \in \mathbb{R}^3$ is the gradient of the function f whose components are the partial derivatives of f.

It should be noted the description of a curve as a smooth geometrical object is not the only one possible, and the selection of functions (5) is ambiguous. Selection of functions $\varphi_1(x)$ and $\varphi_2(x)$ mostly limited by regularity condition [Fradkov et al. (1999)] implying that Jacobian matrix

$$\Upsilon(x) = \begin{bmatrix} \nabla\varphi_1 \times \nabla\varphi_2 \\ \nabla\varphi_1 \\ \nabla\varphi_2 \end{bmatrix} \tag{7}$$

is not degenerate for any vector x belonging to curve S, i.e.

$$det\Upsilon(x) \neq 0.$$

Path following control problem is posed as a problem of maintaining the holonomic relationships between the system outputs set in (5). It is augmented by the description of desired longitudinal motion of the point of the center of mass of the rigid body along the desired path S usually set using the reference velocity of longitudinal motion $V^* = \dot{s}^*$

Consider the errors of the path following [Fradkov et al. (1999), Kapitanyuk and Chepinsky (2013)]. Violation of condition (5) is characterized by deviations

$$e_1 = \varphi_1(x) \tag{8}$$

$$e_2 = \varphi_2(x) \tag{9}$$

zeroed at manifold S.