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Generalized Minimum Projection Method and Its Application to Two Wheeled Mobile Robot Control

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Abstract: In this paper, we propose a generalized minimum projection method considering finitely many layers, desingularization and the stabilization toward the set. Moreover, we propose two global CLFs: with orientation and without orientation of the robot by generalized minimum projection method. Then, we propose a controller based on the proposed CLFs. The effectiveness of the proposed controllers is confirmed by computer simulation.

Keywords: nonlinear control, discontinuous control, Lyapunov methods, asymptotic stability, robot control

1. INTRODUCTION

The minimum prjection method is a control Lyapunov function desgin method for asymptotic stabilization on manifolds Nakamura et al. (2009) Nakamura et al. (2010a) Nakamura et al. (2010b) Nakamura et al. (2013).

In this paper, we propose a generalized minimum projection method considering infinitely many layers, desingularization and the stabilization toward the set.

Two-whleeled mobile robot is an example of nonholonomic systems. Many control strategies are proposed, and a few control Lyapunov functions are also proposed. However, those CLFs are designed based on the Brockett integrator; the CLFs are defined locally and no global CLFs is proposed. Furthermore, stabilization taking into account orientation control of the robot and one without considering orientation is not discussed sufficiently.

In this paper, we propose two global CLFs: with orientation and without orientation of the robot by generalized minimum projection method. Then, we propose a controller based on the proposed CLFs.The effectiveness of the proposed controllers is confirmed by computer simulation.

2. PRELIMINARIES

This paper considers a CLF design problem on manifolds. In this section, we introduce differentiable manifolds and control systems defined on differentiable manifolds.

2.1 Differentiable manifolds Lang (2000); Lee (2003)

In this paper, X denotes a finite-dimensional smooth manifold with dimension n, T_xX is a vector space called the tangent space to X at x. For each $x \in X$, there exists a local chart (W, η) such that $W \subset X$ and $\eta : W \to \Xi = \mathrm{Im}(\eta) \subset \mathbb{R}^n$ is a homeomorphism. Then, $\eta(x)$ is called the local coordinate representation of x with the chart (W, η).

Consider a function $V : X \to \mathbb{R}$ and a chart (W, η) for X. Then, the function $V_W : \Xi \to \mathbb{R}$ defined by $V_W(x) = V \circ \eta^{-1}(x)$ is called the coordinate representation of V. Note that V_W is defined on the subset of $\mathbb{R}^n$. Accordingly, addition and scalar multiplication are available as usual. By the same discussion, f_W denotes the local coordinate representation of $f \in T_xX$. Then, we can define the differential of functions on manifolds. Let $V : X \to \mathbb{R}$ be a smooth function. The mapping $dV : X \to T^*_xX$ denotes the differential of V. Let $(\xi_1, \dots, \xi_n)$ be local coordinates of X with a local chart (W, η). Then, the mapping dV_W can be defined by

$$dV_W(\eta(x)) = \sum_{i=1}^{n} \frac{\partial V_W}{\partial \xi_i}(\eta(x))d\xi_i. \tag{1}$$

The natural pairing $\langle dV, f\rangle$ between a cotangent vector and a tangent vector is defined by Lie derivative as follows: $\langle dV, f\rangle := L_fV$. In local coordinates,

$$\langle dV(x), f(x)\rangle_W = \sum_{i=1}^{n} \frac{\partial V_W}{\partial \xi_i} f_{Wi}(\eta(x)). \tag{2}$$

Local semiconcavity (with linear modulus Cannarsa and Sinestrari (2004)) is defined as follows: a continuous function V is called locally semiconcave at $x \in X$ if there exist $C > 0$ and a sufficiently small neighborhood Ω of x such that

$$V(x) + V(y) - 2V_W\left(\frac{1}{2}(\eta(x) + \eta(y))\right) \le C\|\eta(x) - \eta(y)\|^2 \tag{3}$$

for all $y \in \Omega \subset W$ with a local chart (W, η). Note that the existence of K and C do not depend on the choice of local charts Nakamura et al. (2009).

2.2 Control systems defined on differentiable manifolds

We consider the following control-affine system on a finite-dimensional arc-connected C^1-differentiable manifold $\mathcal{X}$:

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$$\dot{x} = f(x) + g(x) \cdot u \tag{4}$$

$$:= f(x) + \sum_{i=1}^{m} g_i(x) \cdot u_i, \tag{5}$$

where $x \in \mathcal{X}$, $u \in F(\mathbb{R}, \mathbb{R}^m); t \mapsto u(t) \in \mathbb{R}^m$. Moreover, mappings f, $g_i : \mathcal{X} \to T_x\mathcal{X}$ are assumed to be locally Lipschitz continuous with respect to x for all $i \in \{1, \ldots, m\}$, and satisfies $f(0) = 0$, where $0 \in \mathcal{X}$ called the origin.

A function $k : \mathcal{X} \to U$ is called a static state feedback. We consider the sample-and-hold solution defined as follows as solutions of (4).

Definition 1. (Partition). Clarke et al. (1997), Malisoff et al. (2003) Any infinite sequence $\pi = \{t_i \in \mathbb{R}_{\geq 0}\}_{i \in \mathbb{Z}_{\geq 0}}$ consisting of numbers $0 = t_0 < t_1 < t_2 < \cdots$ with $\lim_{i\to\infty} t_i = +\infty$ is called a partition, and the number $d(\pi) := \sup_{i \in \mathbb{Z}_{\geq 0}} (t_{i+1} - t_i)$ is called its diameter.

Definition 2. Clarke et al. (1997), Cortés (2008), Malisoff et al. (2003) Let $u = k(x)$ be a given feedback, π a partition, and $x \in \mathcal{X}$ an initial state. The sample-and-hold solution $\psi(t, x, k(x)) : \mathbb{R}_{\geq 0} \times \mathcal{X} \times U \to \mathcal{X}$ for (4) is defined as the continuous function obtained by recursively solving

$$\dot{x}(t) = f(x(t), k(x(t_i))) \tag{6}$$

from the initial time t_i to the maximal time

$$s_i = \max\{t_i, \sup\{s \in [t_i, t_{i+1}] | x(\cdot) \text{ is defined on } [t_i, s)\}\}, \tag{7}$$

where $x(0) = x$.

We consider sample stability defined as follows Malisoff et al. (2003):

Definition 3. (sample stability). Consider system (4). A feedback $k : \mathcal{X} \to \mathcal{U}$ is said to sample stabilize the origin of the system (4) if the following holds for arbitrary sets $\mathcal{R}_1, \mathcal{R}_2 \in \mathfrak{P}$ such that $\mathcal{R}_1 \subset \mathcal{R}_2$, where $\mathfrak{P}$ denotes the set of all open precompact subset of $\mathcal{X}$ containing the origin.

(1) There exists a set $\mathcal{M} \subset \mathcal{X}$ depending only upon $\mathcal{R}_2$ and two positive numbers $\Omega, T > 0$ depending on $\mathcal{R}_1$ and $\mathcal{R}_2$ such that, for any initial value $x \in \mathcal{R}_2$, for any partition π of the diameter less than Ω, the corresponding sample-and-hold solution $\psi(t, x, k(x))$ satisfies the following conditions:
   (a) $\psi(t, x, k(x)) \in \mathcal{R}_1$ for all $t \geq T$,
   (b) $\psi(t, x, k(x) \in \mathcal{M}$ for all $t \geq 0$.
(2) For each $\mathcal{E} \in \mathfrak{P}$, there exists a set $\mathcal{P} \in \mathfrak{P}$ such that if $\mathcal{R}_2 \subset \mathcal{P}$, $\mathcal{M}$ in (1) can be chosen satisfying $\mathcal{M} \subset \mathcal{E}$.

3. LOCALLY SEMICONCAVE CONTROL LYAPUNOV FUNCTIONS

In this paper, we consider stabilization based on control Lyapunov functions. To define CLF, we introduce the disassembled differental of a locally semiconcave function.

Definition 4. (Disassembled differential). Suppose that $V : \mathcal{X} \to \mathbb{R}$ is a locally semiconcave function. Then, the following set-valued map $\tilde{D}V : \mathcal{X} \to 2^{T_x\mathcal{X}}$ is said to be a disassembled differential of V:

$$\tilde{D}V(x) = \left\{ d\tilde{V}_s(x) \middle| s \in \{s \in \mathcal{S} | V(x) = \tilde{V}_s(x)\} \right\}. \tag{8}$$

The disassembled differential can be written as follows in the local chart $(\mathcal{W}, \eta)$:

$$\tilde{D}V_{\mathcal{W}}(x) = \left\{ \frac{\partial \tilde{V}_{s\mathcal{W}}}{\partial \xi}(\eta(x)) d\xi \middle| s \in \{s \in \mathcal{S} | V(x) = \tilde{V}_s(x)\} \right\}. \tag{9}$$

Then, the locally semiconcave practical control Lyapunov function (LS-PCLF) is defined as follows:

Definition 5. (LS-PCLF). A locally semiconcave practical control Lyapunov function (LS-PCLF) for system (4) is a locally semiconcave function $V : \mathcal{X} \to \mathbb{R}$ such that the following properties hold:

(A1) V is proper; that is, the set $\{x \in \mathcal{X} | V(x) \leq L\}$ is compact for every $L > 0$.
(A2) V is positive definite; that is, $V(0) = 0$, and $V(x) > 0$ for all $x \in \mathcal{X} \backslash \{0\}$.
(A3) For arbitrary R_1, $R_2 \in \mathbb{R}_{>0}$ such that $R_2 > R_1 > 0$, there exists a compact set $\bar{\mathcal{U}} \subset \mathcal{U}$, a positive real constant Q, a discontinuous mapping $p(x) \in \tilde{D}V(x)$, and a local chart $(\mathcal{W}, \eta)$ such that

$$min_{u \in \bar{\mathcal{U}}} \langle p(x), f(x, u) \rangle < -Q,$$
$$\forall x \in \{x | R_1 \leq V(x) \leq R_2\}. \tag{10}$$

With LS-PCLS and disassembled differential the following theorem regarding the Rifford–Sontag-type Rifford (2002), Sontag (1989) controller design holds:

Theorem 1. Assume that V is a locally semiconcave practical control Lyapunov function for (4). Consider a static state feedback control $u = k(x)$ such that

$$k_i(x) = \begin{cases} -\dfrac{\langle p, f \rangle + \sqrt{\langle p, f \rangle^2 + \left(\sum_{i=1}^m \langle p, g_i \rangle^2\right)^2}}{\sum_{i=1}^m \langle p, g_i \rangle^2} \langle p, g_i \rangle \\ \qquad \left(\displaystyle\sum_{i=1}^m \langle p, g_i \rangle^2 \neq 0 \right) \\ 0 \\ \qquad \left(\displaystyle\sum_{i=1}^m \langle p, g_i \rangle^2 = 0 \right), \end{cases} \tag{11}$$

where $p(x) \in \tilde{D}V_{\mathcal{W}} : \mathcal{X} \to T_x^*\mathcal{X}$ is a discontinuous mapping satisfying condition (A3").

Then, if $d(\pi)$ is sufficiently small, (11) sample-stabilizes the origin of the control system (4) in the sample-and-hold sense.

4. GENERALIZED MINIMUM PROJECTION METHOD

4.1 Sheaves on Manifolds

The minimum projection method (Nakamura et al. (2009), Nakamura et al. (2010a), Nakamura et al. (2010b)) is an LS-PCLF design method for control systems on manifolds. In this section we propose a generalized minimum projection method in the manner of "Sheaf Theory." We introduce an old definition of sheaf (Goldblatt (1984), see also MacLane and Moerdijk (1992)) as follows:

Definition 6. (Sheaf). Consider two manifolds X and $\tilde{X}$. Suppose that a mapping $\phi : \tilde{X} \to X$ is locally homeomorphic (étale); i.e., for every $\tilde{x} \in \tilde{X}$ there exists a neighborhood $D \subset \tilde{X}$ of $\tilde{x}$ such that $\phi|_D : D \to \text{Im}(\phi|_D) \subset X$ is a homeomorphism.