Measurement 68 (2015) 310–327

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Structural crack damage detection using transfer matrix and state vector

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ARTICLE INFO

Article history:
Received 12 May 2014
Received in revised form 3 March 2015
Accepted 6 March 2015
Available online 19 March 2015

Keywords:
Cracked beam
Damage detection
Transfer matrix
State vector
Successive identification
Particle swarm optimization

ABSTRACT

A novel structural damage detection scheme is presented here using Transfer Matrix (TM). This scheme is suitable for local crack identification in large structures, by different measurement strategies for the initial state vector near the zone of interest. TM for the damaged element is developed from the theory of fracture mechanics. The state vector at a node consists of displacements, forces and moments at that node which may have internal and external contributions. When this state vector is multiplied with the TM, the state vector at the adjacent node is obtained. The initial state vector at a node is formed by measuring the forces and displacements at that node; the state vectors of the adjacent nodes are predicted by TM with crack parameters included. Measurement strategies for obtaining the initial state vector involving strain gauges and accelerometers are important aspect of this paper. The mean square error between measured and TM predicted responses is minimized using a non-classical heuristic algorithm. The stiffness integrity index for the lumps mass system, and crack depth and location of beam structures are the optimization variables. Each element is identified using TM successively with either complete or incomplete set of displacement measurements. Numerically simulated studies of a twelve story lumped mass system, a cantilever and a sub-structure of nine member frame structure are presented here. The scheme is also experimentally validated by identification of cracks in a sub-structure of a fixed beam. The speed and accuracy of this method are compared with other existing method and found to be good.

1. Introduction

The process of detecting damage and monitoring the condition of the structure is known as Structural Health Monitoring (SHM). It is very important for civil, mechanical, aerospace structures, since they undergo repeated cyclic loading that may cause an unexpected fatigue failure even if there is a small crack. Generally the cracks in the structures are monitored by visual inspection and some of non-destructive methods such as ultrasonic methods, optical methods, radiography, magnetic field methods, eddy-current methods and thermal field methods. Ambu et al. [1] investigated the feasibility of using optical methods to detect impact damage in thin laminates. Specimens were analysed by two different techniques such as holographic, electronic speckle pattern interferometry and proved that holographic technique is superior to the other one. Kessler et al. [2] presented a damage detection technique with the application of lamb wave and found that this technique promising more information about the damage presence and its severity than other methods. Nath et al. [3] detected and sized the damage in the complex geometry welds using ultrasonic time of flight

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http://dx.doi.org/10.1016/j.measurement.2015.03.010

diffraction technique. Nair and Cai [4] proposed a typical continuous bridge monitoring system using acoustic emission evaluation technique. Kaphle et al. [5] explains that acoustic emission technique is best suited for monitoring bridges rather than using visual inspection methods. However, all of these methods require the vicinity of the damage is known a priori and the structural element to be inspected is accessible. Hence, the vibration based crack detection methods will be a good alternative for SHM.

The vibration based damage detection algorithms are categorized into two types. *i.e.* frequency domain and time domain algorithms. In the former, the change in natural frequency and mode shape are determined from which the damage parameters are estimated [6,7]. The latter detects the crack parameters from the measured time domain signals such as acceleration, velocity, displacement at selected nodes of the structure [8,9]. Since it does not require any further transformation, the measured signals can be used directly.

The application of TM for eigenvalue problems is reviewed here. Liew et al. [10] used TM in the field of rotor dynamics for transient response analysis of non-symmetrical unbalance flexible rotor. Ghasemalizadeh et al. [11] determined natural frequency and plotted FRF for multi rotor system supported on two bearings using TM. Khiem and Lien [12] determined natural frequency of beam with multiple cracks using the TM method. The crack element is considered as a hinge with no mass and length, but has only flexibility. The intact portions of the beam is connected by the crack element. The overall TM for the entire structure is obtained by series multiplication of TMs corresponds to the intact and crack portions of the structure.

To the best of authors' knowledge, Nandakumar and Shankar [13] were the first to identify structural parameters of a ten DOF lumped mass system and a cantilever using TM as an inverse problem. Further, they developed an improved consistent mass based TM (CMTM) and used for structural parameter identification and noted the advantages of speed, accuracy and convenience of local identification for large structures. Later Nandakumar and Shankar [14] developed a new TM for structural elements with damping, identified structural parameters successfully. The innovative aspect of this paper is that TM for a cracked beam element is developed and is used for crack damage detection. This method is novel because its essence is to measure the state vector at the initial (starting) location close to the damage zone, and predict the damages in the adjacent elements by successive multiplications of the TM. An important aspect of this paper is the experimental measurement strategy involving strain gauges and accelerometers to estimate the initial state vector.

A brief review of crack Finite Element (FE) formulations is presented here. A crack in a beam element provides significant local flexibility which is the function of crack depth and location. The crack affects the natural frequency of the structure as well as its dynamic behavior. Gounaris and Dimarogonas [15] developed an elemental compliance matrix for cracked prismatic beam element by assuming the crack increases the flexibility of the element due to strain energy concentrations in the vicinity of the crack tip under load. The stiffness and mass matrices for the cracked beam element have been developed from the compliance matrix and forced vibration of a cantilever with open crack was studied. Ibrahim [16] formulated another new stiffness matrix using the TM approach for the beam element with Elasto-plastic crack. The crack was modelled as torsional spring without mass and has flexibility equal to the additional flexibility due to the crack in the element. Chondros et al. [17] developed a continuous cracked beam theory for Euler–Bernoulli beams with single and double edge crack. The crack was modelled as a continuous flexibility using the displacement field in the vicinity of the crack. Sinha et al. [18] formulated another new stiffness matrix for the cracked beam element with equivalent varying flexural rigidity near the crack. This approach performs good only at the lower modes of vibration. Zheng and Kessissoglou [19] developed an overall stiffness matrix from the overall additional flexibility matrix rather than local additional flexibility matrix by which a better natural frequency and mode shape of structures were found. Krawczuk et al. [20] developed new FE matrices for cracked beam elements using Elasto-plastic fracture mechanics. Two different polynomial functions are assumed for the two portions of the element on both sides of the crack. FE procedure was applied with suitable mathematical relations at the crack. Viola et al. [21] developed a new prismatic beam element with crack and the effect of crack in stiffness and mass matrices was investigated.

Some studies which pertain to crack identification in structures are briefly presented here. Gounaris et al. [22] formulated a compliance matrix for a cracked beam element by which cracks were identified in a cantilever by measuring coupled responses. However, it is suitable for only one edge crack in the beam and requires many nomograms for each frequency of vibration. Abdalla et al. [23] used linear matrix inequality method for detecting damage in structural members. However this method could not find the exact location and magnitude of damage. Krawczuk [24] detected the crack in a beam like structures using a wave propagation method with Genetic Algorithm (GA) and a gradient search technique. Recursive least-squares estimation with unknown inputs approach is proposed to identify the structural parameters, such as the stiffness, damping, and other non-linear parameters, as well as the unmeasured excitations [25]. An adaptive tracking technique is also implemented to track the variations of structural parameters due to damages.

Lee [26] identified multiple cracks in a cantilever from vibration amplitudes using Newton–Raphson method and singular value decomposition method. Since classical optimization technique is used and number of cracks are unknown, a good initial guess for the crack parameters is necessary, otherwise there is a chance of convergence in local optima. Vakil-Baghmisheh et al. [27] identified the crack parameters in a cantilever by minimizing the error in measured and predicted values of first four natural frequencies using GA. Gao and Lu [28] used residual generation approach using measured acceleration response for damage detection in structural members. Tee et al. [9]