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Passification based signal-parametric adaptive controller for agents in formation ⋆

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Abstract: In the paper the novel method for ensuring an overall formation stability for the case of significant variations of the agents parameters in formation is proposed. The local agent controller is designed by employment of the signal-parametric adaptive strategy, based on the passification method. The approach is demonstrated by an example of altitude control of the quadrocopter formation. The simulation results for a group of 25 quadrocopters are given, showing efficiency of the suggested method.

Keywords: formation control, quadrocopter, adaptive control, passification method, sliding mode,

1. INTRODUCTION

In the recent decades development of control of agents in formation, which can interact autonomously with the environment and other agents to perform various tasks, which are beyond the ability of individual agent, retains growing attention. Research on multi-agent formations attracts increasing interest during the last decade due to various potential applications, such as platoons of satellites, unmanned aerial vehicles (UAVs), mobile robots, autonomous underwater vehicles, automated highway systems, etc. Particularly, a serious attention is paid to the development of consensus protocols for multi-robotic systems (Porfiri et al., 2007; Ni and Cheng, 2010; Li et al., 2013). Researches in the field of multi-copter formations control are usually inspired by the recent results in the area of coordinated control for multi-agent systems, including the control of coordinated motion for group of autonomous mobile agents (Jadbabaie et al., 2003; Fax and Murray, 2004) and the consensus problem (Olfati-Saber et al., 2007).

Since the multi-agent formation is a system of the interconnected dynamical plants, ensuring stability of the overall high-order system is an important and challenging task. Stability condition for formations of linear time invariant (LTI) plants may be significantly simplified for the case of the agents with identical dynamics. The well known result of such a kind is obtained by Fax and Murray (2004), who reduced the stability analysis of the multi-agent formation to checking stability condition for a single agent taking into account the spectrum of the Laplace matrix, describing the information flow graph of the formation. The limitation of this result lies in the assumption that the agents are LTI systems with identical dynamics. In the numerous papers the approaches are presented to overcome this limitation. For example, Fradkov and Junussov (2011); Dzhunusov and Fradkov (2009) considered the problem of adaptive synchronization for networks of interconnected dynamical subsystems. Decentralized algorithms of adaptive control are designed and the conditions of synchronizability are found by the method of passification and the Kalman-Yakubovich-Popov lemma (Fradkov, 2003).

In the present paper the novel approach based on the passification method and the signal-parametric adaptive control is employed for designing the formation control, ensuring approximate identical dynamics of the each closed-loop agent in the formation for the wide area of the particular agents parameter values. What is more, the prescribed dynamical properties of the LTI agents in the formation may be secured by the mentioned result by Fax and Murray (2004), which could not be possible at the case of non-identical agents dynamics.

The rest of the paper is organized as follows. Application of the *passification method* to the field of the adaptive control is briefly recalled in Sec. 2, where the signal-parametric adaptive control law with implicit reference model and its modified version are presented. Multi-agents formation control with local signal-parametric adaptive controllers is considered in Sec. 3. Section 4 presents the application example of control of altitude for a group of 25 quadrocopters. Concluding remarks and the future work intentions are given in Sec. 5.

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2. SIGNAL-PARAMETRIC ADAPTIVE CONTROL

Following (Fradkov and Andrievsky, 2007; Andrievskii and Fradkov, 2006) let us recall design of the variable-structure controllers (Utkin, 1992) and the signal-parametric adaptive controllers with the *implicit reference model* (Andrievskii et al., 1988; Stotsky, 1994; Andrievskii et al., 1996; Druzhinina et al., 1996) based on the passification method (Fradkov, 1974, 1980; Fradkov et al., 1999; Fradkov, 2003).

Consider the following linear time-invariant (LTI) plant model:

$$\dot{x} = Ax + Bu, \quad y = Cx, \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^l$, A, B, C are the matrices of appropriate dimensions. Let the control aim be $\lim_{t\to\infty} x(t) = 0$. Introduce the auxiliary aim as ensuring the *sliding mode* motion over the surface $\sigma = 0$, where $\sigma = Gy$, and G is the given $l \times n$.

Let us consider the following control law:

$$u = -\gamma \operatorname{sign} \sigma, \quad \sigma = Gy, \tag{2}$$

where gain $\gamma > 0$ is a design parameter. It is proven in (Andrievskii et al., 1988) that this aim is reached for system (1), (2) if there exist a matrix $P = P^{\mathrm{T}} > 0$ and vector K_* such that $PA_* + A_*^{\mathrm{T}} P < 0$, $PB = GC$, $A_* = A + BK_*^{\mathrm{T}} C$. As follows from passification theorem (Fradkov, 1974, 2003), these conditions are satisfied only if the transfer function $GW(\lambda)$, where $W(\lambda) = C\big(\lambda I_n - A\big)^{-1} B$, is hyper-minimumphase and the sign of the high-frequency transfer gain GCB is positive. If these conditions are satisfied, then for a sufficiently large gain γ one gets that $\lim_{t\to\infty} x(t) = 0$. To eliminate dependence of the system stability on the initial conditions and the plant model parameters, the following "signal-parametric" (or "combined") adaptive control law was developed (Andrievskii et al., 1988, 1996):

$$u = -K^{\mathrm{T}}(t)y(t) - \operatorname{sign} \sigma(y), \quad \sigma(y) = Gy \tag{3}$$

$$\dot{K}(t) = \sigma(y)\Gamma y(t), \tag{4}$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0$ and $\gamma > 0$ are the matrix and scalar gains of the algorithm.

It is proven (Fradkov, 1990; Andrievskii et al., 1996), that $\sigma(t)$ converges to zero in a finite time for any bounded domain of the initial states of (1), (4). Following lines of the proof, given in (Fradkov, 1990), it may be shown that in the case of $l = 1$ the following modified signal-parametric adaptive controller may be used instead of (4) ensuring the sliding-mode motion on the surface $\sigma \equiv 0$:

$$u = -k^{\mathrm{T}}(t)y(t) - \gamma_\sigma \operatorname{sign}(\sigma(y))\sqrt{|\sigma(y)|}, \ \sigma(y) = Gy \tag{5}$$

$$\dot{k}(t) = \gamma y(t)^2, \tag{6}$$

where gains $\gamma > 0$, $\gamma_\sigma > 0$ are design parameters. The sign term in (5) is borrowed from (Emelyanov et al., 1986; Levant, 1993), producing the smoother control signal than that in (3). Signal σ_t may be referred to as the "*adaptation error*" since equivalence $\sigma_t \equiv 0$ means fulfillment of the auxiliary (adaptation) aim.

The adaptation algorithms like (4), (6) are rarely used in practice due to the fact that the controller gain $K(t)$ grows indefinitely under the action of external disturbances or in the presence of sensor errors. For avoiding this, various methods of robustification (Fradkov, 1980, 1990) (see also (Tsakalis, 1992)) were developed, among which introduction of the *parametric feedback* and introduction of the *dead zone* are the basic ones.

Parametric feedback regularization leads to the following adaptation algorithm (instead of (5)):

$$u = -k(t)y(t) - \gamma_\sigma \operatorname{sign}(\sigma(y))\sqrt{|\sigma(y)|}, \ \sigma(y) = Gy \tag{7}$$

$$\dot{k}(t) = \gamma y(t)^2 - \alpha\big(k(t) - k^0\big), \quad k^0 = k(0), \tag{8}$$

where coefficient $\alpha \geq 0$ is the design parameter, introducing a feedback in the adaptation law. The above method of regularization is applicable also if there is discretization on time in the case of the digital control, unmodelled plant dynamics and smooth nonlinearity in the control loop, cf. (Derevitsky and Fradkov, 1974; Ljung, 1977; Seron et al., 1994).

3. FORMATION OF AGENTS WITH LOCAL SIGNAL-PARAMETRIC ADAPTIVE CONTROLLERS

Let us recall the stability condition for formation of identical LTI agents (which are assumed to be asymptotically stable) (Fax and Murray, 2004, Theorem 4): it is necessary and sufficient that the Nyquist plot of the open-loop system does not encircle the points $-\lambda_i^{-1}$ on the complex plane for all $i = 1, \ldots n$, such that $\lambda_i \neq 0$, where λ_i, $i = 1, 2, \ldots, N$ are the eigenvalues of the Laplacian L of the information flow graph. The similar condition may be also applied to discrete-time systems, see (Fax and Murray, 2004) for more details.

An application example of this result for evaluation stability of the networked control for quadrocopters formation is presented in (Tomashevich and Andrievsky, 2014), where the way for improving the system quality is also proposed.

Consider a group of N SISO agents whose dynamics models have the following state-space representation:

$$\begin{aligned} \dot{x}^j(t) &= A^j x^j(t) + B^j u^j(t), \\ y^j(t) &= C^j x^j(t), \ z^j(t) = D^j x^j(t), \end{aligned} \tag{9}$$

where $j = 1, \ldots, N$ is the sequence number, of the agent in the formation, $x^j(t) \in \mathbb{R}^n$ denotes the state vector of jth agent, $u^j(t) \in \mathbb{R}^1$ is a scalar local control signal, acting to the jth agent, $y^j(t) \in \mathbb{R}^1$ is the scalar output signal, used for formulation of the control aim, $z^j(t) \in \mathbb{R}^l$ is the vector of measured variables, A^j, B^j, C^j, D^j are the matrices and vectors of appropriate dimensions.

Let the control aim be tracking the external reference signal $r^j(t)$ with the prescribed dynamics. To this end let us apply the signal-parametric adaptive control law (7), (8), modifying it to the purpose of tracking. Define the sliding manifold as $G^j z^j(t) + g_0^j r^j(t) \equiv 0$, where row vector $G^j \in \mathbb{R}^{1\times l}$ and scalar g_0 are the design parameters, which should be found for ensuring the prescribed tracking dynamics on the sliding mode, see (Utkin, 1992) for more details. This leads to the following expression of the adaptation error $\sigma^j(t)$, introducing the reference signal to the implicit reference model:

$$\sigma^j(t) = G^j z^j(t) + g_0^j r^j(t). \tag{10}$$

Then the signal-parametric (local) control law for each jth agent in the formation is as follows: