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Parameter identification of a discretized biased noisy sinusoidal signal ☆

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A B S T R A C T

A discrete time, on-line approach is proposed for the simultaneous identification of the frequency, the amplitude and the phase parameters in continuous noisy sinusoidal signals. The proposed technique takes the exact discretization of a continuous sinusoidal signal and generates exact computation formulas for the frequency, bias, amplitude and phase parameters, calculated in terms of quotients involving functions of the output and of some of its delayed values. The formula yields exact parameter calculations in the noiseless case, and it can be synthesized by means of time-varying filters whose outputs are obtained on-line. When the treated signal is affected by additive noises, a least squares-based "invariant filtering" is proposed, resulting in an enhancement of the signal to noise ratio of the factors conforming the basic algorithm. The scheme is proven and analysed by means of a laboratory experiment, a noise analysis and a comparison against other existing methods, where the proposed method shows accurate results with fast convergence rates.

1. Introduction

The problem of parameter estimation in sinusoidal waves has attracted much attention due to its importance in theoretical studies and real-world applications, from areas as civil engineering [1,2], robotics and automatic control [3,4], bio-medics [5,6], signal processing and power systems [7,8] among others. It should be emphasized that numerous approaches have been developed for frequency estimation in sinusoidal signals ranging from Time domain-based schemes, Kalman filtering to nonlinear adaptive estimation (see [9–11]). Some works include innovative theoretical comparisons with traditional techniques (see [5]). A rather complete survey examining different approaches and performance of the algebraic methods with many other previously proposed approaches can be found in the articles by Trapero et al. [12,13].

Fast (non-asymptotic) continuous time algorithm for algebraic identification of the frequency parameter in continuous sinusoidal signals have been successfully tested, and compared, among other competitive approaches in [13,14]. The algebraic approach considers the problem as an exact computation formula for the unknown parameters when interpreted in a noiseless continuous time-domain environment. When suitably combined with classical low-pass filters in an "invariant manner" an enhancement is obtained of the quality of the algorithm response against additive measurement noises. This approach achieves good results when the algorithm is provided by a sufficient quantity of signal samples. However, the case of parameter

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identification with a poor quantity of supplied samples implies a different treatment, considering a discrete time case. The advantage of sinusoidal waves is the fact that the function describing their behaviour in discrete time (without bias) is given in terms of an exact second order difference equation, in which the discrete-time equivalent identification approach can be implemented, and the external facts as the constant bias can be solved by means of algebraic manipulations.

In this article, a discrete-time fast identification approach for continuous signals is developed and tested in a laboratory experiment dealing with a test sinusoidal signal. The methodology can be applied in hardware restriction processes where a fast frequency computation is required. Concretely, this paper deals with the problem of estimating the angular frequency ω, the bias parameter b, the amplitude A, and the phase ϕ of a sinusoidal signal of the form $x(t) = A\sin(\omega t + \phi) + b + \xi(t)$, $\xi(t)$ an additive noisy signal, by means of exact discretization. This scheme allows us to transform a continuous sinusoidal signal into a third order difference equation. The algebraic estimation approach is applied on the difference equation and by means of algebraic manipulations the parameters of the signal are obtained as a function in terms of the output and of its immediate delayed terms. This methodology is based on the fast identification method of discrete-time linear systems used in [15] for the area of servo-vision control.

Some numerical simulations were done to test the methodology behaviour under different noisy signals, and a comparison test was carried out to compare this technique with another fast estimation method. A laboratory experiment is carried out for the frequency estimation, under noisy measurements, of a sampled sinusoidal signal. The proposed approach is suitably combined with invariant filtering aimed at reducing the signal to noise ratio. The bias, amplitude and the phase parameters are simultaneously calculated once the angular frequency is accurately determined.

The paper is organized as follows: In Section 2, the difference equation of a noiseless biased sinusoidal signal is obtained, and the identification approach is proposed for the noiseless case. Sections 3 and 4 consider the case of a biased and noisy signal case, proposing the invariant filtering approaches for the parameter identification process, as well as the procedure for the amplitude and phase parameters identification. In Section 5, the methodology is illustrated with some numerical simulations; to set the efficiency of the approach in different circumstances, this section tests the identification method with different signal to noise ratios and sampling frequencies, using a percentage error index. Section 6 deals with some comparisons with other fast and effective classic discrete time frequency identification procedures. Section 7 shows some experimental results and, finally, some conclusions and suggestions for further work are collected in Section 8.

2. Difference equation of a single sinusoidal wave

Let $x(t)$ be a continuous sinusoidal signal of the form

$$x(t) = A\sin(\omega t + \phi) \tag{1}$$

where $A \in \mathbb{R}$, is the amplitude, $\phi \in \mathbb{R}$, is a constant phase component and $\omega \in \mathbb{R}$, denotes the angular frequency of $x(t)$. This continuous signal is sampled with a period T yielding the following expression:

$$x(kT) = x(t)|_{t=kT} = A\sin(\omega kT + \phi)$$

By applying well known trigonometrical identities we obtain:

$$\begin{aligned} x(kT - 2T) &= A\sin(\omega kT - \omega T - \omega T + \phi) \\ &= \cos(\omega T)x(kT - T) - A\sin(\omega T)\cos(\omega(kT \\ &\quad - T) + \phi) \end{aligned} \tag{2}$$

Using the fact that $\cos(\theta_1)\sin(\theta_2) = 1/2(\sin(\theta_2 + \theta_1) + \sin(\theta_2 - \theta_1))$ and substituting in (2), we obtain after simple manipulations:

$$x(kT) = 2\cos(\omega T)x(kT - T) - x(kT - 2T) \tag{3}$$

where (3) is a difference equation which expresses (1) in an exact discretization form. For the sake of simplicity, define $\mu = \cos(\omega T)$. Thus, the estimation of μ is expressed as

$$\hat{\mu}(kT) = \frac{x(kT - 2T) + x(kT)}{2x(kT - T)} = \frac{n_\omega(kT)}{d_\omega(kT)}$$

and

$$\hat{\omega}(kT) = \frac{1}{T}\arccos\left(\frac{n_\omega(kT)}{d_\omega(kT)}\right)$$

That is, the value of ω is directly obtained by some measurements of the output signal and its delayed terms.

3. A biased and sinusoidal signal

Consider now a biased sinusoidal signal with an exact discretization process:

$$x_b(kT) = A\sin(\omega kT + \phi) + b \tag{4}$$

with $b \in \mathbb{R}$ being an additive offset component. $x(kT)$ can be written as follows:

$$x(kT) = x_b(kT) - b$$

Using the relation between x and x_b in (3) leads to the following expression:

$$2\mu(x_b(kT - T)) = x_b(kT - 2T) + x_b(kT) - 2b(1 - \mu) \tag{5}$$

Applying a time-shift operation in (5) and subtracting to eliminate the constant term we have:

$$\begin{aligned} 2\mu(x_b(kT - T) - x_b(kT - 2T)) &= x_b(kT) - x_b(kT - T) \\ &\quad + x_b(kT - 2T) - x_b(kT - 3T) \end{aligned}$$

The frequency can be then estimated as follows:

$$\begin{aligned} \hat{\mu} &= \frac{1}{2}\frac{x_b(kT) - x_b(kT - T) + x_b(kT - 2T) - x_b(kT - 3T)}{x_b(KT - T) - x_b(KT - 2T)} \\ &= \frac{n_{\omega b}(kT)}{d_{\omega b}(kT)} \end{aligned}$$

i.e.,

$$\hat{\omega}(kT) = \frac{1}{T}\arccos\left[\frac{n_{\omega b}(kT)}{d_{\omega b}(kT)}\right] \tag{6}$$