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IFAC-PapersOnLine 48-11 (2015) 280–285

Sensorless Nonsmooth $\mathcal{H}_\infty$-Tracking Synthesis of Servosystems with Backlash and Coulomb Friction

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Abstract: The nonsmooth phenomena such as backlash and Coulomb friction, often occurring in mechanical systems, typically produce undesired inaccuracies, oscillations and instability thereby degrading the system performance. In this paper, the $\mathcal{H}_\infty$ synthesis is developed for a class of nonsmooth systems and it is then applied to an industrial emulator where the described phenomena are present, and an elastic band is included in the transmission motor-load, and only motor position measurements are feasible. The effectiveness of the proposed method is supported by experimental results.

Keywords: Backlash; nonsmooth $\mathcal{H}_\infty$-control; Coulomb friction; output-feedback.

1. INTRODUCTION

Backlash is a common phenomenon in mechanical and hydraulic systems, which occurs due to the mechanical play between adjacent movable parts. The backlash phenomenon is accompanied with hysteresis between the input and output positions and it is thus represented as a nonlinear dynamic system with delays, oscillations and impacts, which are well-known to produce inaccuracy, limit cycles, early wear of the mechanical parts involved, and hence to result in poor performance of the system.

In order to deal with an adequate representation of the transmitted torque with backlash, the deadzone model from (Nordin et al., 1997; Lagerberg and Egardt, 2003), which includes flexibility in the transmission or gear, is chosen for the present investigation. Robust tracking of servosystems with backlash and dry friction (another common nonsmooth nonlinearity that degrades the performance of mechanical systems) is the primary concern of this work.

Robust $\mathcal{H}_\infty$ synthesis of systems with backlash may be found in (Acho et al., 2013) where the standard linear $\mathcal{H}_\infty$ approach is utilized while treating backlash nonlinearities as bounded disturbances. The nonlinear $\mathcal{H}_\infty$ synthesis is applied in (Aguilar et al., 2007) to a servosystem regulation problem where the backlash phenomenon is modeled using an approximate model whereas (Bentsman et al., 2013) solves the regulation problem for a specific system with backlash in the input using the deadzone model. The available literature on control of systems with backlash focuses on the stabilization and regulation problems where the Coulomb friction is ignored and very few results (see, e.g., (Lin et al., 1996; Marton and Lantos, 2009; Friedland, 1997)) are available where the regulation of systems is subject to backlash and Stribeck/Coulomb friction effects. This motivates further investigation on the tracking synthesis, successfully treating backlash and dry friction phenomena in combination. Such a problem, becomes even more challenging if the full information on the system is no longer available that corresponds to a practical situation where no load measurements are normally feasible.

The main results of the paper are summarized as follows

- Extend the output feedback nonsmooth $\mathcal{H}_\infty$ control synthesis to tracking of servomechanisms with dry friction and backlash, and with an elastic band, included in the motor-load transmission.
- Encoders are usually attached to actuators, therefore include an extra encoder at the load side becomes unnecessary. Here, we consider that only motor position measurements are available.

The underlying load tracking problem is solved by means of an appropriate static motor reference trajectory model that results in the desired load tracking. The nonsmooth $\mathcal{H}_\infty$ synthesis is then developed to asymptotically track the nominal motor motion and to attenuate external disturbances. Since the standard nonlinear $\mathcal{H}_\infty$ approach, as in (Basar and Bernhard, 1995; Isidori and Astolfi, 1992; Van Der Schaft, 1992), does not admit a straightforward

[⋆] Y. Orlov gratefully acknowledges the financial support from CONACYT (Consejo Nacional de Ciencia y Tecnología) under Grant 165958.

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Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.09.198

application under existing nonsmooth effects the nonsmooth $\mathcal{H}_\infty$ synthesis, recently proposed in (Orlov and Aguilar, 2014), is invoked to deal with a nonsmooth representation of the system backlash and dry friction. To enhance the controller performance a dynamic filter of the motor reference trajectory is additionally involved into the closed-loop system and its influence on the system performance is experimentally studied in a laboratory testbed. Achieving good performance of the synthesized servosystem motion is the main contribution of this paper.

The rest of the paper is outlined as follows. In Section 2, the generic nonsmooth $\mathcal{H}_\infty$ synthesis is presented. Section 3 introduces a servosystem with backlash for which the load tracking problem is stated and resolved via motor position feedback $\mathcal{H}_\infty$ synthesis. Section 4 conducts an experimental study of an industrial emulator to support the theory. Finally, Section 5 collects some conclusions.

2. PRELIMINARIES: GENERIC NONSMOOTH $\mathcal{H}_\infty$ SYNTHESIS

For later use, the local $\mathcal{H}_\infty$ synthesis is recalled from (Orlov and Aguilar, 2014) for a generic nonautonomous nonsmooth system of the form

$$\begin{aligned} \dot{x}(t) &= f(x(t),t) + g_1(x(t),t)w(t) + g_2(x(t),t)u(t), \\ z(t) &= h_1(x(t),t) + k_{12}(x(t),t)u(t), \\ y(t) &= h_2(x(t),t) + k_{21}(x(t),t)w(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the disturbance attenuator, $w(t) \in \mathbb{R}^r$ is the unknown disturbance, $z(t) \in \mathbb{R}^l$ is the output to be controlled, $y(t) \in \mathbb{R}^p$ is the available measurement of the system.

The nonsmooth $\mathcal{H}_\infty$-control problem for a generic system (1) is to find a locally stabilizing output feedback controller

$$\begin{aligned} u &= \mathcal{K}(\xi,t) \\ \dot{\xi} &= \mathcal{F}(\xi,y,t), \end{aligned} \tag{2}$$

with internal state $\xi \in \mathbb{R}^s$ such that the $\mathcal{L}_2$-gain of the closed-loop system (1), driven by (2), is locally less than γ. Solving the above problem under γ approaching the infimal achievable level γ^* in (3) yields a (sub)optimal $\mathcal{H}_\infty$-controller with the (sub)optimal disturbance attenuation level γ^* $(\gamma > \gamma^*)$.

For convenience of the reader recall that the generic system (1) locally possesses $\mathcal{L}_2$-gain less than γ iff the following inequality holds

$$\int_0^T \|z(t)\|^2 dt < \gamma^2 \int_0^T \|w(t)\|^2 dt \tag{3}$$

for all $T > 0$, for all the system trajectories initialized in the origin, and for all piecewise continuous functions $w(t) \in \mathcal{L}_2(0,T)$ (particularly, constant disturbances are admitted) such that the state trajectories remain in a vicinity of the origin.

The following assumptions are imposed on the generic system (1):

A1. The functions $f(x,t)$, $g_1(x,t)$, $g_2(x,t)$, $h_1(x,t)$, $h_2(x,t)$, $k_{12}(x,t)$, $k_{21}(x,t)$ are continuous in t, and locally Lipschitz continuous in x for all t;

A2. For almost all $t \in \mathbb{R}$, there exists a neighborhood $U_t(0)$ of the origin $x = 0$, possibly dependent on t, such that the functions, listed in Assumption A1, are uniformly bounded in t, twice continuously differentiable in x, and their first and second order state derivatives are piecewise continuous and uniformly bounded in $t \in \mathbb{R}$ for all $x \in U(0)$.

A3. $h_1^T(x,t)k_{12}(x,t) = 0$, $k_{12}^T(x,t)k_{12}(x,t) = I$, $k_{21}(x,t)g_1^T(x,t) = 0$, $k_{21}(x,t)k_{21}^T(x,t) = I$ for all t.

A4. $f(0,t) = 0$, $h_1(0,t) = 0$, and $h_2(0,t) = 0$ for all t.

Assumption A1 admits nonsmooth nonlinearities guaranteeing the well-posedness of the above dynamic system under integrable exogenous inputs. Assumption A2, made for a technical reason, allows one to locally linearize the closed-loop system, driven by a smooth feedback controller. Assumption A3 is a simplifying assumption inherited from the standard $\mathcal{H}_\infty$-control problem; relaxing this assumption is indeed possible but it would greatly complicate the formulas to be worked out. Assumption A4 ensures that the origin is an equilibrium point of the non-driven $(u = 0)$ disturbance-free $(w = 0)$ dynamical system (1).

The local synthesis involves the standard linear $\mathcal{H}_\infty$-control problem for the linearized system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B_2(t)u(t) \\ z(t) &= C_1(t)x(t) + D_{12}(t)u(t) \\ y(t) &= C_2(t)x(t) + D_{21}(t)w(t) \end{aligned} \tag{4}$$

where

$$\begin{aligned} &A(t) = \frac{\partial f}{\partial x}(0,t), \quad B_1(t) = g_1(0,t), \quad B_2(t) = g_2(0,t), \\ &C_1(t) = \frac{\partial h_1}{\partial x}(0,t), \quad C_2(t) = \frac{\partial h_2}{\partial x}(0,t), \\ &D_{12}(t) = k_{12}(0,t), \quad D_{21}(t) = k_{21}(0,t). \end{aligned} \tag{5}$$

Using the time-varying version (Orlov and Aguilar, 2014) of the real bounded lemma (Brogliato et al., 2007), one can note that the following conditions are necessary and sufficient for a solution of the linear $\mathcal{H}_\infty$-control problem to exist:

C1. there exists a positive constant ε_0 such that the the perturbed Riccati equation

$$\begin{aligned} -\dot{P}_\varepsilon(t) &= P_\varepsilon(t)A(t) + A^T(t)P_\varepsilon(t) + C_1^T(t)C_1(t) \\ &+ P_\varepsilon(t)\left[\frac{1}{\gamma^2}B_1B_1^T - B_2B_2^T\right](t)P_\varepsilon(t) + \varepsilon I, \end{aligned} \tag{6}$$

possesses a positive definite symmetric solution $P_\varepsilon(t)$ for each $\varepsilon \in (0,\varepsilon_0)$;

C2. while being coupled to (6), the perturbed Riccati equation

$$\begin{aligned} \dot{Z}_\varepsilon(t) &= A_\varepsilon(t)Z_\varepsilon(t) + Z_\varepsilon(t)A_\varepsilon^T(t) + B_1(t)B_1^T(t) \\ &+ Z_\varepsilon(t)\left[\frac{1}{\gamma^2}P_\varepsilon B_2B_2^TP_\varepsilon - C_2^TC_2\right](t)Z_\varepsilon(t) + \varepsilon I \end{aligned} \tag{7}$$

possesses a positive definite symmetric solution $Z_\varepsilon(t)$ for each $\varepsilon \in (0,\varepsilon_0)$ with $A_\varepsilon(t) = A(t) + \frac{1}{\gamma^2}B_1(t)B_1^T(t)P_\varepsilon(t)$.

Equations (6) and (7) are now utilized to derive a local solution of the nonsmooth $\mathcal{H}_\infty$-control problem for system (1). Under partial state measurements, the local $\mathcal{H}_\infty$ synthesis is augmented with a dynamic compensator running in parallel. The compensator is derived by means of the perturbed Riccati equations (6) and (7), that appear