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Accelerometer errors in the measurement of dynamic signals

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ABSTRACT

The paper presents the way of determining the time dependent characteristics of maximum error generated by an accelerometer in case it measures dynamically changing signals of vibrations. For that purpose, a domain has been specified, which may contain the class of signals maximizing the error criterion assumed. Indispensable constraints have been imposed on signals, which result from the dynamic properties of the accelerometer examined. These constraints concern amplitude and rate of change. Constraint values have been determined using a mathematical model of an accelerometer obtained from concurrent measurements of its amplitude and phase characteristics. The error has been calculated in relation to a standard, having the form of an ideal low-pass filter. The characteristics of the error as a function of measurement time have been provided. The numerical solutions presented in the article refer to a popular accelerometer of the PCB type, and error in the form of integral square-error.

1. Introduction

The results of measurements of vibrations constitute a principal source of information in the widely understood investigations of mechanics and the behaviour of orogen, geophysics, safety of engineering structures on the surface, or miners working in underground mines. Vibrations are a result of propagation and reflection of seismic waves, which can be generated on the surface or under the ground. Because of the type of signal generated, those sources can be divided into impulse sources, generating short signals with high amplitude (sledgehammer, weight drop, sparker, betsy gun, buffalo gun), as well as vibratory sources (seismic vibrators) generating long signals with small amplitudes. Vibrations may be caused by random sources resulting directly from human activity (cars, undergrounds, tunnelling, under-ground mining) as well as natural causes, independent from humans (earthquakes) [1,2]. Vibrations are usually measured on the ground's surface, at specific measuring points, using measuring instruments designed specifically for that purpose, called accelerometers, and the measurements are then, via analog to digital converters and data acquisition cards, registered on a computer disc [3,4]. Measurements are taken either along a 1D axis only, vertical to the surface of the ground – component Z – or simultaneously in 3D, in directions perpendicular to one another: component Z, as well as a horizontal component having a North–South direction (N–S) and a horizontal component having an East–West direction (E–W). Usually, acceleration is measured or after integration in data acquisition cards, the vibration speed of the surface on which the accelerometer has been placed is measured. The correct measurement condition demands that all harmonics of the measured signal are within the range of constant gain of the transmission frequency range of the accelerometer, whereas the phase of each of them is linearly delayed. This condition, in reference to measurement instruments meant for dynamic measurements, has been applied in practice for many years, in line with valid legal regulations [5–8]. Most

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often the measurement of the amplitude-frequency characteristic was sufficient in this case [9].

The problem lies in the fact that, knowing the transmission frequency range of the accelerometer, we do not know the shape of the signal of the vibration measured, thus we do not know how many harmonics it contains, nor their specific amplitudes, frequencies, and phase shifts. It is customary to assume that harmonics of the measured signal are within the amplitude-frequency range characteristics of the accelerometer, but it does not have to be the case, as various sources generate vibrations in completely different ranges of harmonic. At the same time, the signals, penetrating various media, different from each other, such as rocks, soils, clays, and water, significantly change their own dynamic properties. In consequence, we cannot say which of the harmonics of the measured signal are within the accelerometer passband frequency, and not knowing the spectrum of that signal we are not able to determine its measurement error [10].

An additional problem arises from the fact, that the accelerometer error should be determined in such a way that it is invariant to the dynamics of signals measured by it. This means that it should be independent of any signal unknown *a priori*, which could be measured by means of that accelerometer [11]. For such assignments, the problem of error determination has not been solved as yet, nor have any regulations been established in that respect. In consequence, for the measurement systems designed for dynamic measurements, no accuracy classes exist, nor procedures for their determination.

The problem of an error meeting the above requirements may be solved only through determination of its maximum value. This means that a signal of any shape, which could occur on the input of the measurement system, will always generate the error of less than or at most equal to this maximum value. This means that the determined error is completely independent of the shape and dynamics of the input signal. In order to determine such an error we shall use the theory of maximum errors [12–16].

This paper deals with the determination of the measurement time dependent characteristics of the maximum error of the vibrations measured by a piezoelectric accelerometer of the PCB type. It presents the principles of determination of maximum error, discusses the assumptions, resulting from its dynamic properties, concerning the set of acceptable maximizing signals as well as presenting the algorithms indispensable for such error calculation. The calculation requires knowledge of the mathematical model of the accelerometer considered, the model of the standard error criterion assumed, the constraints imposed on the form of the acceptable signals, and the measurement time [17]. The successive steps, which enable the solving of this problem are presented below.

2. Model of the accelerometer

Let us present a commonly known model of an accelerometer

$$K(s) = \frac{\alpha}{s^2 + 2\beta f_0 s + f_0^2} \tag{1}$$

in parametric form, by means of a product of two vectors: $\boldsymbol{\lambda}$ and $\boldsymbol{\Theta}$. The λ vector contains the parameters α, f_0 and β separated from model (1), while the $\boldsymbol{\Theta}$ vector contains the relations referring to the independent variable f. We thus have

$$K(f) = \frac{1}{\lambda_0 + \lambda_1 jf - \lambda_2 f^2} = \frac{1}{\boldsymbol{\lambda\Theta}(f)} \tag{2}$$

or

$$K(f) = \mathrm{A}(f)\exp[j\Phi(f)] \tag{3}$$

where $\alpha/f_0^2, f_0, \beta$ are amplification coefficients, non-damped natural frequency and damping factor respectively; while $A(f)$ and $\Phi(f)$ present the modulus and phase of transfer function $K(f)$. In Eqs. (2) and (3)

$$\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \lambda_2] = \left[\frac{f_0^2}{\alpha}, \frac{\beta f_0}{\alpha}, \frac{1}{\alpha}\right] \tag{4}$$

$$\boldsymbol{\Theta}^T(f) = [1, j2f, -f^2] \tag{5}$$

The parameters of the mathematical model Eq. (1), presented in Eq. (4), are usually determined on the basis of its amplitude characteristics, ignoring the impact of phase characteristics. We shall present below the method of simultaneous use of both characteristics to determine those parameters, which creates the possibilities of a much more accurate representation of the accelerometer studied. We shall apply the weighted method of least squares, utilizing data from the simultaneous measurement of amplitude and phase characteristics [18].

For that purpose, we determine a $2N \times 3$ element matrix $\mathbf{P}$ and vector $\mathbf{V}$ resulting from the measurement points of the frequency characteristics

$$\mathbf{P} = \begin{bmatrix} \mathrm{Re}\left[\Theta_0^T(f_0)\right] & \mathrm{Re}\left[\Theta_1^T(f_0)\right] & \mathrm{Re}\left[\Theta_2^T(f_0)\right] \\ \mathrm{Re}\left[\Theta_0^T(f_1)\right] & \mathrm{Re}\left[\Theta_1^T(f_1)\right] & \mathrm{Re}\left[\Theta_2^T(f_1)\right] \\ \vdots & \vdots & \vdots \\ \mathrm{Re}\left[\Theta_0^T(f_{N-1})\right] & \mathrm{Re}\left[\Theta_1^T(f_{N-1})\right] & \mathrm{Re}\left[\Theta_2^T(f_{N-1})\right] \\ \mathrm{Im}\left[\Theta_0^T(f_0)\right] & \mathrm{Im}\left[\Theta_1^T(f_0)\right] & \mathrm{Im}\left[\Theta_2^T(f_0)\right] \\ \mathrm{Im}\left[\Theta_0^T(f_1)\right] & \mathrm{Im}\left[\Theta_1^T(f_1)\right] & \mathrm{Im}\left[\Theta_2^T(f_1)\right] \\ \vdots & \vdots & \vdots \\ \mathrm{Im}\left[\Theta_0^T(f_{N-1})\right] & \mathrm{Im}\left[\Theta_1^T(f_{N-1})\right] & \mathrm{Im}\left[\Theta_2^T(f_{N-1})\right] \end{bmatrix} \tag{6}$$

$$\begin{aligned} \mathbf{V}^T = \{ & \mathrm{Re}\ [\mathrm{A}(f_0)^{-1}\exp(-j\Phi(f_0))] \\ & \mathrm{Re}\ [A(f_1)^{-1}\exp(-j\Phi(f_1))] \ldots \\ & \mathrm{Re}\ [A(f_{N-1})^{-1}\exp(-j\Phi(f_{N-1}))] \\ & \mathrm{Im}\ [A(f_0)^{-1}\exp(-j\Phi(f_0))] \\ & \mathrm{Im}\ [A(f_1)^{-1}\exp(-j\Phi(f_1))] \ldots \\ & \mathrm{Im}\ [A(f_{N-1})^{-1}\exp(-j\Phi(f_{N-1}))]\} \end{aligned} \tag{7}$$

where N denotes the number of measurement points.

For the above relations, vector $\boldsymbol{\lambda}$ of the model in Eq. (2) is calculated from the formula