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The Effect of the Local Filter Adjustment on the Accuracy of Federated Filters

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Abstract: The efficiency of federated filters without reset of local filters is studied. A comparative analysis of additional loss in the accuracy of such filters with respect to the optimal centralized filter is performed. The conditions for adjustment of local filters providing minimum additional loss in accuracy are determined. Analytical solution is illustrated by methodic examples. An example of a federated filter application to an integrated navigation system is considered.

Keywords: accuracy, estimation, Kalman filter.

1. INTRODUCTION

Currently, it has become common practice that the state of dynamical systems, in particular, during navigation signal processing, is estimated both by centralized data processing and decentralized methods (Carlson 1995, Tupysev 1998, Maier et al. 2010). A characteristic feature of centralized data processing methods in the linear statement of the estimation problem is processing of all measurements with one centralized Kalman filter (KF). In this case, the state vector contains both the state vector describing the behavior of the dynamic system and the state vectors of filters used to describe nonwhite measurement errors of all sensors. It is significant that centralized KF provides the optimal estimate of the state vector with a minimum covariance matrix of the estimate error in real time (Gelb et al. 1974, Brown et al. 1997,). Decentralized data processing, or the so-called federated filters, are used for modular architecture of systems. They involve the use of a bank of local filters included in the measurement module for primary processing of their measurements. In such filters, global parameters are generated by fusion of the local estimates obtained from different sensors in the master filter.

Decentralized methods have been the subject-matter of a large number of publications both on theoretical studies (Carlson 1995, Tupysev 1998) and practical applications. For example, the federated filtering methods are used for aiding airborne navigation systems with the data from a GPS receiver, altimeter, radar, and Doppler velocity sensor (Maier et al. 2012), for designing navigation and control systems for quadrocopters (Benzerrouk 2014), and integration of marine inertial navigation systems (Stepanov et al .2013).

Federated filters can be conditionally divided into two groups: no-reset federated filters (NRFF) and federated filters with reset local filters. A feature characteristic of the first group is using the information generated only in one measurement module to calculate the prediction parameters in local filters. The second group uses global parameters—estimates and the calculated covariance matrix—to reset local filters before calculating the prediction parameters, which actually means information redistribution between sensors.

Of vital importance in applied estimation is not only to estimate parameters, but also to obtain their accuracy characteristics. In particular, these characteristics are very important for navigation data processing (Kulakova and Nebylov 2008, Kulakova et al. 2010, Loparev et al. 2014). In KF filtering algorithms, accuracy is estimated with the use of a covariance matrix (Gelb 1974, Brown et al. 1997). Since, in general, federated filters are not optimal, the covariance matrix calculated in such a filter cannot be an adequate accuracy measure for the global estimate of the system state. It is therefore very important to provide the so-called guaranteed properties of federated filters, which lie in the fact that the calculated covariance matrix of the estimate error is the upper bound for the real covariance matrix of the estimate error (Loparev et al. 2009) This property, in turn, makes it possible to use the calculated covariance matrix as an accuracy measure for the global estimate (Tupysev 1999).

It has been shown before that guaranteed properties of federated filters can be provided under certain conditions of local filter adjustment. It should be noted that the conditions providing guaranteed estimation allow for some freedom in choosing the adjustment of local filters. However, the recommendations for the adjustment parameters given in (Tupysev 1998, Hongmei Zhang 2007, Maier et al. 2010, Deng Hong et al. 2012, Binglei Guan et al. 2012, Xiong Zhi et al. 2013,) are not theoretically justified and refer mainly to federated filters with reset local filters with the same state vectors. The research presented in this paper is aimed at analyzing the adjustment conditions of NRFF providing guaranteed estimation and at estimating additional loss in accuracy for white-noise and nonwhite-noise measurement errors, with respect to the optimal centralized filter.

Notice that, as a rule, the federated filtering methods are considered in the discrete statement of the estimation problem, but in this paper, it is convenient to make a

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comparative analysis of centralized and federated filters for the case of continuous measurements. The research is conducted by analyzing simplest methodical examples, which allow additional loss of accuracy to be estimated analytically; also given is an example of data processing in an integrated navigation system.

2. OPTIMAL ESTIMATION WITH A CENTRALIZED FILTER AND FEATURES OF ESTIMATION USING FEDERATED FILTERING METHODS

Consider the solution of the estimation problem with a centralized KF and NRFF in a rather general statement, when the behavior of a dynamical system is described by the equation

$$\dot{X}_0(t) = F_0(t)X_0(t) + \xi_0(t), \quad (1)$$
$$X_0(0) \in N\{0, P_0(0)\}, \xi_0(t) \in N\{0, Q_0(t)\}$$

using the measurements from L sensors:

$$z_i(t) = H_{0i}(t)X_0(t) + H_{Ci}(t)C_i(t) + v_i(t), \quad (2)$$
$$v_i(t) \in N\{0, R_i(t)\}, i \in \overline{1, L},$$

in which nonwhite-noise measurement errors are described by the equations:

$$\dot{C}_i(t) = F_{Ci}C_i(t) + \xi_{Ci}(t), \quad (3)$$
$$C_i(0) \in N\{0, P_{0ci}(0)\}, \xi_{Ci}(t) \in N\{0, Q_{Ci}(t)\},$$

where $Q_0(t)$, $Q_{ci}(t)$ $R_i(t)$ are the known matrices of the noise power spectral density (PSD).

For simplicity, it is assumed that $Q_0(t)$ is not a degenerate matrix and that parameters $\xi_{Ci}(t), \xi_0(t), \nu_i(t), C_i(0), x_0(0)$ are not correlated with each other.

For convenience, index t is omitted in the cases when the time-dependence of the parameters is evident.

Introducing vector $X_P = \left[X_0^T, C_1^T, C_2^T ... C_N^T\right]^T$, we can write model (1), (2), (3) as

$$\dot{X}_P = F_P X_P + \xi_P, Z_P = H_P X_p + v_P, v_P \in N\{0, R_P\}, \quad (4)$$

where $F_P = diag\{F_0, F_{C1}, F_{C2} ... F_{CL}\}$,

$$Q_P = diag\{Q_0, Q_{C1}, Q_{C2} ... Q_{CL}\}, v_P = \left[v_1^T, v_2^T ... v_L^T\right]^T,$$

$$H_P = \begin{bmatrix} H_{01} & H_{C1} & 0 & ... & 0 \\ H_{02} & 0 & H_{C2} & ... & 0 \\ ... & ... & ... & ... & ... \\ H_{0L} & 0 & 0 & ... & H_{CL} \end{bmatrix}, \xi_P = \left[\xi_0^T, \xi_{C1}^T, \xi_{C2}^T ... \xi_{L2}^T\right]^T. \quad (5)$$

It is known that in this problem statement the optimal estimate of the state vector X_0 with a minimum covariance matrix can be obtained using the centralized KF shown in Fig. 1, with the following parameters: F_p, Q_p, H_p, R_p.

Notice that in the KF implementation, P_P is the real covariance matrix of the vector X_P and, consequently, P_0 for the vector X_0 is the real covariance matrix.

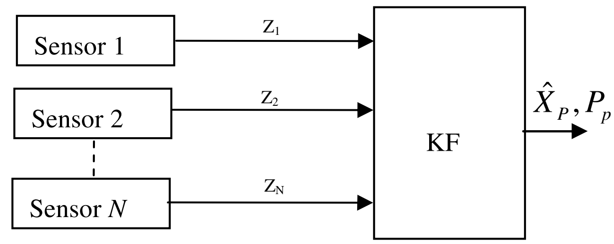

Fig.1. Centralized filter.

Now consider the application of the federated filter methods as applied to the above statement, in the presence of nonwhite measurement errors. The essence of these methods is using a bank of Kalman-type filters (Fig. 2) to obtain local parameters with subsequent generation of global parameters by their averaging. As applied to the case of continuous measurements, local filters estimate state vector $X_i = \left[X_{0i}^T, C_i^T\right]^T$ for the model

$$\dot{X}_{0i} = F_0 X_{0i} + \xi_{0i}; \; \dot{C}_i = F_{Ci}C_i + \xi_{Ci}, X_{0i}(0) \in N\{0, P_{0i}(0)\}, \quad (6)$$

$$z_i = H_{0i}X_{0i} + H_{Ci}C_i + v_i, v_i \in N\{0, R_i\}, \xi_{0i} \in N\{0, Q_{0i}\}, \quad (7)$$

using the FK procedures with $H_i = \left[H_{0i} \quad H_{Ci}\right], F_i = diag\{F_0, F_{Ci}\}, Q_i = diag\{Q_{0i}, Q_{Ci}\}$,

and matrices $Q_{0i}(0)$, $P_{0i}(0)$ are such that

$$\sum_{i=1}^{N} P_{0i}^{-1}(0) = P_0^{-1}(0), \quad \sum_{i=1}^{N} Q_{0i}^{-1}(t) = Q_0^{-1}(t). \quad (8)$$

As this takes place, the global parameters—the estimate of state vector X_0 and matrix P_0 —are obtained by weighted averaging of the parameters from the local filters using the equations

$$P_K = (\sum_{i=1}^{N} P_{0i}^{-1})^{-1}, \; \hat{X}_K = P_K \sum_{i=1}^{N} P_{0i}^{-1}\hat{X}_{0i}, \quad (9)$$

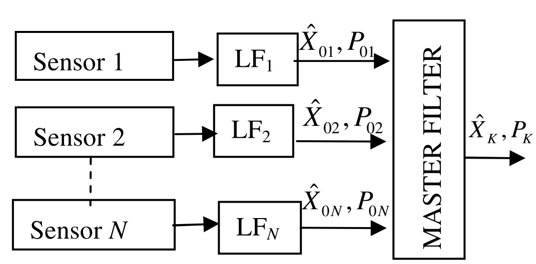

Fig. 2. Federated filter.

Notice that the global estimate $\hat{X}_K$ obtained with (9) is generally not optimal, and matrix P_K is not the error covariance matrix of estimate $\hat{X}_K$. However, it can be shown that if the conditions for adjustment of local filters (8) are satisfied, the federated filters without reset of local filters will have a guaranteed estimation property in the sense that the inequality below holds: $D_K \le P_K$, in which D_K is the real