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An H_∞ Technology for Control of the Integrity of the Kalman Type of Estimating Filters with the Use of Adaptive Robust Procedures

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Abstract: This paper is devoted to the problem of increasing the level of confidence when the state of nonlinear dynamical systems (DSs) is estimated with the help of an extended Kalman filter (EKF). The problem mentioned involves a need for the prevention of EKF divergence. The proposed solution of the above problem relies both on the technique of matrix inequalities and on the technology of H_∞ optimization. Compared with a traditional EKF, the solution obtained includes, in addition, the loops intended to monitor and protect the integrity of an estimating filter. The results of the mathematical simulation are given, which corroborate the effectiveness of the proposed approach.

Keywords: H_∞ optimization, Kalman filter, monitoring, adaptation

1. INTRODUCTION

At present, the application of an extended Kalman filter (EKF) to the estimation of errors of nonlinear dynamical systems (DSs) is considered to be justified, see Maybeck (1982). The technology of estimation relies on a mathematical description of the functioning of both the reference (unperturbed) DS and an actual (perturbed) DS. The ideal vector *Y(t)* and the actual vector $Y_r(t)$ of state parameters are made to correspond to such DSs; these vectors are described by the following differential equations:

for the ideal DS: $dY(t)/dt = \dot{Y}(t) = F[Y(t)]$; (1)

for an actual DS: $\dot{Y}_r(t) = F[Y_r(t)] + G(t)\xi(t)$, (2)

where $\xi(t) = [\xi_1(t)\ldots\xi_l(t)]^T$ is the vector of disturbances that affect the DS, which is characterized by the covariance matrix $M[\xi(t)\xi^T(t-\tau)] = Q(t)\delta(t-\tau)$; $\delta(t-\tau)$ is the delta-function; $M[\ldots]$ is the operator of mathematical expectation; $G(t)$ is the matrix of disturbance intensities.

Parameters of the ideal DS and the actual DS are related by the following error equation:

$dx(t)/dt = \dot{x}(t) = A(t)x(t) + G(t)\xi(t)$, (3)

where $x(t) = \Delta Y(t) = Y_r(t) - Y(t)$ is the vector of DS errors; $A(t) = \partial F[Y(t)]/\partial Y\big|_{Y(t)=Y_r(t)}$ is the matrix of coefficients that characterize the dynamics of variation of DS errors.

The estimates $\hat{x}(t)$ of DS errors are obtainable through the use of the EKF by processing the following observations:

$$z(t) = h[Y_r(t)] - h[Y(t)]_{SEI}, \quad (4)$$

where $h[Y(t)]_{SEI}$ is an observed value formed by the sensor of information that is external (SEI) with respect to the DS, and the above observed value has the model $h[Y(t)]_{SEI} = h[Y(t)] + \vartheta(t)$;

$\vartheta(t)$ is the vector of perturbations in a measuring channel, which has the covariance matrix

$M[\vartheta(t)\vartheta^T(t-\tau)] = R(t)\delta(t-\tau)$.

In the EKF, the interrelation of observations (4) and DS errors is taken into account via the following mathematical model:

$$z(t) = H(t)x(t) + \vartheta(t), \quad (5)$$

where $H(t) = \partial h[Y(t)]/\partial Y\big|_{Y(t)=Y_r(t)}$ is the matrix for the relation of observed parameters and the vector of DS errors.

An observable dynamical system with the EKF in an error estimation loop can be represented by a diagram shown in Fig. 1,

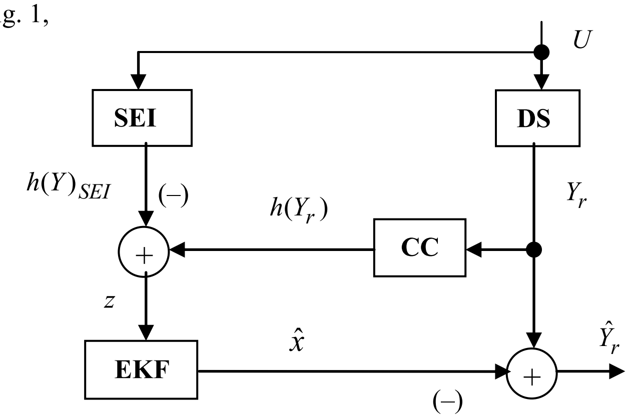

Fig. 1. Block diagram of an observable DS with the EKF

where U is the vector of control actions; CC is a coordinate converter. It should be noted that as a SEI, use can be made of the mathematical model of the reference DS.

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It is known, see Fitzgerald (1971), that the inaccuracy of models (3), (5) and other causes that are methodical and instrumental in character result in the divergence of the EKF. The EKF divergence manifests itself in the fact that the actual estimation errors $\delta_j = x_j - \hat{x}_j$ considerably differ from their predicted mean square values $\sigma_j = \sqrt{P_{jj}}$ obtained upon solving the Riccati equation for the covariance matrix *P*. Here $\hat{x}_j$ is an estimate of the *j*-th component of the vector x_i. It should be noted that the actual estimation errors come to light only in the stage of mathematical simulation.
Compensation for the "a priori" uncertainty of mathematical models is possible on the basis of adaptive, see Chin (1979), and guaranteeing, see Gelig et al. (2008), approaches.

The purpose of this paper is to increase the information reliability of estimation procedures for the parameters of observable DSs on the basis of integration of guaranteeing and adaptive approaches to the protection of EKFs from divergence.

2. AN H_∞ TECHNOLOGY FOR PROTECTION THE INTEGRITY OF AN ESTIMATING FILTER

The level of confidence in estimating the vector of DS errors can be increased when an EKF output test is organized. One possible approach to the solution of such a problem is based on the employment of the norm of the transfer function *H(p)*, see Polyak (2014), from the EKF output disturbances ξ_i, ϑ_i to the estimation errors δ_i, i.e.,

$$\|H(p)\|_\infty = \sup_{\xi,\vartheta\neq 0} \frac{\|\delta\|_2^2}{\|\xi\|_2^2 + \|\vartheta\|_2^2}, \tag{6}$$

where $\|\xi\|_2 = (\sum_{i=i_0}^{\infty} \xi_i^T \xi_i)^{1/2}$ is the Euclidian norm for the vector ξ_i. The Euclidian norms for the vectors δ_i and ϑ_i are determined in a similar way; sup denotes the upper bound.
For stable estimation, the following test condition can be formed:

$$\|H(p)\|_\infty \le \gamma^2, \tag{7}$$

where $\gamma^2 \ge 0$ is the parameter that can be considered as a tolerance. In practical applications, the interval of H_∞ optimization is bounded by the time $i_0 \le i \le i_f$ of DS functioning, and it is assumed that $\|\delta\|_2^2 < \infty$; $\|\xi\|_2^2 < \infty$; $\|\vartheta\|_2^2 < \infty$. It should be noted that test condition (7) can be represented in an explicit form, only when accurate observations directly connected with estimation errors are at hand. In practice, the vector δ_i of actual estimation errors, as a rule, is not known. Because of this, expression (6) is transformed to the form that implicitly allows for the above-mentioned errors. This can be done, provided that from the predicted estimates $\hat{x}_{i/i-1} = \Phi_i \hat{x}_{i/i-1}$ the "a priori" residuals

$$\nu_{i/i-1} = z_i - H_i \hat{x}_{i/i-1} \tag{8}$$

are determined, and from the updated estimates $\hat{x}_{i/i-1}$ (after processing the residuals $\nu_{i/i-1}$) the "a posteriori" residuals

$$\eta_{i/i} = z_i - H_i \hat{x}_{i/i} \tag{9}$$

are determined, where Φ_i is a transition matrix for the vector of DS errors, which is determined from the solution of the differential equation $\dot{\Phi}(t) = A(t)\Phi(t, t_{i-1})$ for $\Phi(t_{i-1}, t_{i-1}) = I$; I is an identity matrix of the appropriate dimension.
Then criterion (6) can be associated with the following its modification:

$$\frac{\gamma^{-2} \sum_{i=i_0+1}^{i_f} \|\eta_{i/i}\|_2^2}{\sum_{i=i_0+1}^{i_f} (\|\xi_i\|_2^2 + \|\nu_{i/i-1}\|_2^2)} \le 1 \tag{10}$$

Mutual correspondence between criteria (7) and (10) can be seen, provided that observations (8) and (9) are represented in an expanded form, namely:

$$\nu_{i/i-1} = H_i x_i + \vartheta_i - H_i \hat{x}_{i/i-1} = H_i \delta_{i/i-1} + \vartheta_i; \tag{11}$$

$$\eta_{i/i} = H_i x_i + \vartheta_i - H_i \hat{x}_{i/i} = H_i \delta_{i/i} + \vartheta_i \tag{12}$$

Following criterion (10), taking account of expressions (11) and (12), an estimation strategy is to be formed, such that the quotient of the norm for the vector $\delta_{i/i}$ of output errors and the norm for the vector $\delta_{i/i-1}$ of input errors is no more than 1.
Inequality (10), which is written in the following equivalent form:

$$J = J_0 - \gamma^{-2} \sum_{i=i_0+1}^{i_f} \|\eta_{i/i}\|_2^2 \ge 0, \tag{13}$$

where $$J_0 = \sum_{i=i_0+1}^{i_f} (\|\xi_{i-1}\|_2^2 + \|\nu_{i/i-1}\|_2^2), \tag{14}$$

in view of the constraint equation

$$\Phi_i x_{i-1} + \Gamma_i \xi_{i-1} - x_i = 0, \tag{15}$$

can be regarded as a quadratic estimation criterion for the vector x_i. The scalar γ^2 is a tunable parameter that maintains the fulfillment of condition (13). Here Γ_i is a transition matrix for the vector of disturbances.