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Echo State Neural Network Based State Feedback Control for SISO Affine Nonlinear Systems

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Abstract: Echo state network (ESTN) is a new recurrent neural networks (RNN) with a simpler training method. Based on ESTN, this paper address a state feedback control algorithm for a class of perturbed SISO nonlinear systems in the affine form. The control algorithm is implemented without *a prior* knowledge of the nonlinear system. The network weights can be tuned on line by the Recursive Least Squares (RLS) method without off line learning phase needed. The convergence and the Bounded Input Bounded Output (BIBO) stability of the ESTN controller are proven. Moreover, all signals involved in the closed loop are proven to be exponentially bounded and then the stability of the system. We have used the tracking problem of one-link rigid robotic manipulator system as an example to verify the effectiveness of the proposed method.

Keywords: Echo state neural network, Direct adaptive control, State feedback control, Nonlinear systems

1. INTRODUCTION

In the past several years, neural network based adaptive control schemes for nonlinear systems have been active research area. Under the assumption that all states of the plant are available for measurement, adaptive neural network state feedback control schemes have been developed as in Ge et al. (1999),Wai (2003), Tang et al. (2006), Belarbi and Chemachema (2007), and Wang et al. (2014). When only the system output can be measured, the observer-based adaptive neural network control were proposed for a certain class of unknown nonlinear systems for example Sridhar and Khalil (2000), Chien et al. (2011),Castaneda and Esquivel (2012), and Chemachema (2012). In the case of multiple input-multiple output nonlinear systems, many adaptive output feedback control schemes were proposed as in Chen et al. (2010), Li et al. (2011) and Kostarigka and Rovithakis (2012).

The multilayer perceptron (MLP) and the RBF neural networks are the most popular topologies used in these schemes, but those available have a static feed forward network structure that requires a large number of neurons. Furthermore, the weight updates of them do not use the internal neural netwok information. To address these drawbacks, recurrent NNs (RNNs) Williams and Peng (1990), Ku and Lee (1995), and Lee and Teng (2000) have been introduced to apply dynamic feedback structures in which the present activation state is a function of the previous activation state and of the present inputs. The recurrent neural networks have been developed in many adaptive control schemes Chin-Min et al. (2012), Li et al. (2014), Chun-Fei (2014), and Michael et al. (2014).

Recently, the echo state network (ESTN) proposed in Jaeger (2001), is an advanced recurrent neural network algorithm. The ESTN is used as dynamic reservoirs with a large number of neurons that are randomly interconnected and/or self connected. The reservoir itself is fixed, once it is selected. During the training process of ESTN, only the output connections are trained through on line linear regression or on line methods, such as the recursive least square (RLS). Unlike other recurrent architectures, the use of random weights in the ESTN and the training algorithm greatly simplify the training of the network and eliminate the issues of stability, convergence, and local minima. The ESNT has been successfully applied for many applications. The ESTN has been developed in predication the chaotic time series Jaeger and Haas (2004), and Zhiwei and Min (2007). In the identification and control of dynamic systems, it has been used as in Jaeger et al. (2007), Xu et al. (2005), Venayagamoorthy (2007), Pan and Wang (2012), Seong and Lee (2013), and Seong and Lee (2014).

This paper addresses the development of the ESTN in the state feedback control scheme for a special class of SISO nonlinear systems. We develop the ESTN with on line updating law to approximate an ideal controller deduced from the certainly equivalent approach. Likewise to the method proposed in Belarbi and Chemachema (2007) and Chemachema (2012), we propose a simple method to

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estimate the control error to derive the updating laws of the neural network controller. Using the estimated control error, the recursive least squares method is used to train the output weights of the ESTN controller. According to the main features of the ESTN, the convergence of the output weights of the controller is easy to prove. Furthermore, the Lyapunov direct method is then used to prove the global exponential boundedness of all the signals involved in the closed loop, and hence the stability of the system. The performance of the proposed scheme is evaluated using one-link rigid robotic manipulator system. The paper is organized as follows. Section (2) describes the structure and the learning of the ESTN. In section (3), we describe the nonlinear system and the problem formulation. The proposed state feedback control based on the ESTN is presented in section (4). The simulation results of the proposed scheme is introduced in section (5). Section (6) provides a summary of the proposed work.

2. THE ECHO STATE NEURAL NETWORK

This section describes the structure and the learning of the ESTN.

2.1 ESTN Structure

The ESTN composes of a hidden layer (dynamical reservoir) with randomly interconnected neurons and a memoryless output layer (readout). Let K, N, L represent the number of input, reservoir, and output units, respectively, $z(t) = [z_1(t), ..., z_K(t)]^T$ are the K-dimensional external input, $v(t) = [v_1(t), ..., v_N(t)]^T$ are the N-dimensional reservoir activation state, and $y(t) = [y_1(t), ..., y_L(t)]^T$ are the L-dimensional output vector at time step t. The state and output equations of the ESTN are

$$v(t+1) = f(\mathbf{W}v(t) + \mathbf{W}^{in}z(t) + \mathbf{W}^{bias}) \qquad (1)$$

$$y(t) = \mathbf{W}^{out}v(t+1) \qquad (2)$$

where $f(.)$ is the activation function (typically tanch functions), $\mathbf{W} \in \Re^{N\times N}$, $\mathbf{W}^{in} \in \Re^{N\times K}$, $\mathbf{W}^{bias} \in \Re^{N\times 1}$ and $\mathbf{W}^{out} \in \Re^{L\times N}$ denote internal connection, input connection, bias, and the output connection weights, respectively. An additional nonlinearity can be applied to $y(t)$ in (2), as well as feedback connection from output $y(t-1)$ to the internal state $v(t)$ in (1).

Definition 1. For an ESTN in (1) trained by using many set of input vector $z(t)$ in compact intervals $\mathcal{Z}$, it has echo state property with respect to $\mathcal{Z}$ if for any infinite sequence $z(t)$, and for all state sequences $\acute{v}(t)$ and $v^{''}(t)$ associated with the reference sequence; i.e.,

$$v^{'}(t+1) = f(\mathbf{W}v^{'}(t) + \mathbf{W}^{in}z(t) + \mathbf{W}^{bias}) \qquad (3)$$

$$v^{''}(t+1) = f(\mathbf{W}v^{''}(t) + \mathbf{W}^{in}z(t) + \mathbf{W}^{bias}) \qquad (4)$$

the following equality holds for a while:

$$v^{'}(t) = v^{''}(t) \qquad (5)$$

This definition states that, if the ESTN starts from two initial states $\acute{v}(0)$ and $v^{''}(0)$ with the same input sequence, after running for long time, the state sequence of the ESTN would converge.

2.2 ESTN Learning

The basic idea of the ESTN's learning is that the values of $\mathbf{W}$, $\mathbf{W}^{in}$, and $\mathbf{W}^{bias}$ are fixed without adaptation. Whereas, only the readout weights, $\mathbf{W}^{out}$, will be adapted. All wights matrices are randomly initialized according to the standard distribution $\sim N(0, 1)$. Let λ_{max}, the largest absolute eigenvalue (called the spectral radius) of $\mathbf{W}$. If $|\lambda_{max}| > 1$, then the network (1) has no echo state property for any $\mathcal{Z}$. Experimental results deduced that if $|\lambda_{max}| < 1$ is a sufficient condition for the echo state property Jaeger (2001). Thus, after initialization, the matrix $\mathbf{W}$ is normalized by dividing it with λ_{max}.
The learning of the output weights $\mathbf{W}^{out}$ in (1) can be expressed as solving a system of linear equations by

$$Y_{target} = \mathbf{W}^{out}V \qquad (6)$$

with respect to $\mathbf{W}^{out}$, where $V \in \Re^{N\times T}$ is all $v(t)$ produced by presenting the reservoir with $z(t)$ in (1), and $Y_{target} \in \Re^{L\times T}$ are all $Y_{target}(t)$, both collected into respective matrices over the training period $t = 1, ..., T$. Equation.(6) can be solved using a method for finding least squares solutions of the optimization problem

$$min_{\hat{W}^{out}} = \left\|\mathbf{W}^{out}V - Y_{target}\right\| \qquad (7)$$

3. PROBLEM FORMULATION

Consider the n^{th} order nonlinear dynamical system expressed in the affine form

$$\begin{aligned} \dot{x_i} &= x_{i+1}, i = 1, 2, ..., n-1 \\ \dot{x_n} &= f(x) + g(x)u + d \\ y &= x_1 \end{aligned} \qquad (8)$$

where $x = [x_1, x_2, ..., x_n] \in \Re^n$, $u \in \Re$, $y \in \Re$ are the state variables, control input, and system output, respectively. d is external bounded disturbance. Equation. (8) can be rewritten in the following state space form

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B(f(\mathbf{x}) + g(\mathbf{x})u + d) \\ y &= C^T\mathbf{x} \end{aligned} \qquad (9)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1 \\ 0 & 0 & 0 & . & . & . & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

In this paper, the following assumption is taken for the system in (8)

Assumption 1. $f(\mathbf{x})$ and $g(\mathbf{x})$ are unknown nonlinear and bounded function, $g(\mathbf{x}) \neq 0$ for all values of $\mathbf{x}$. The output is required to follow a bounded smooth trajectory y_r, d is bounded disturbance and the derivatives of y_r up to n^{th} order (i.e.,$y_r^{(n)}$) exist and bounded.

Define the desired output and the tracking error vector as

$$\begin{aligned} \mathbf{y}_r &= [y_r, \dot{y_r}, ..., y_r^{(n-1)}] \in \Re^n \\ \mathbf{e} &= \mathbf{y}_r - \mathbf{x} \\ e_1 &= y_r - x_1 \end{aligned}$$