Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-11 (2015) 360–365

On Stabilization of Switched Linear MI Systems Using in Part Common Left Eigenvector Assignment Based on LMIs

D. Krokavec, A. Filasová, V. Serbák

Department of Cybernetics and Artificial Intelligence, Technical University of Košice, Košice, Slovakia (e-mail: dusan.krokavec@tuke.sk, anna.filasova@tuke.sk, vladimir.serbak@tuke.sk)

Abstract: The problem of assigning in part common left eigenvector assignment by the state feedback control of linear multi-input systems is considered in the paper. Based on linear matrix inequalities characterization, the exposition of the problem is formulated to handle a common left eigenvector for switched modes. Limited by preexisting degrees of freedom, which factually makes mutually contingent subsets of the closed-loop eigenvalue spectrum of each mode, the design conditions for in part common left eigenvector assignment of all modes are derived in the terms of linear matrix inequalities. The procedure provides a way available in linear switched control system design while the state-space control structures take standard forms. An example is included to assess the properties of the technique and its application.

Keywords: switched linear systems, eigenvector assignment, state control, matrix forms.

1. INTRODUCTION

Interesting in design of the state feedback control for switching linear systems, the stability of such systems gives rise to a number of mathematical problems (Shorten et al. (2007), Sun and Ge (2011)), since switching has to be concerned with the verifiable conditions that guarantee the system exponential stability for a switching signal. Since the system matrices of the modes set have to be Hurwitz, a large number of methods, relying on the construction of common quadratic functions for modes, was proposed. For details on quadratic methods application in the switched systems design see, e.g., Liberzon (2003) and Sun and Ge (2005). An improvement, which could mitigate this weakness, was proposed in Kouhi and Bajcinca (2011b). This topic exploits the partial eigenstructure assignment principle (Duan and Wang (2003)), giving possibility for adaptation to state-dependent switching linear systems synthesis.

Eigenstructure assignment belongs to the salient design problems of modern control theory, concerning with the establishment of eigenvalues and associated eigenvectors via feedback control laws to achieve prescribed closed-loop specifications. To obtain the required control gain matrices, notable papers offered algorithms using the explicit parametric form (Fahmy and O'Reilly (1982)), Sylvester's equation (Bhattacharyya and de Souza (1982)), singular matrix decomposition properties (Andry et al. (1983), Klein and Moore (1985)), minimal condition number of eigenvector matrix (Kautsky et al. (1985)), Moore's parametric form (Schmid et al. (2011)), etc. The use of these algorithms become wide-spread in the control systems literature, and primary texts, among many others, are given by Kailath (1980) and Fairman (1998). Moreover, recent advances have resulted in the use of linear matrix inequality (LMI) techniques to solve the eigenstructure assignment problem (Duan and Yu (2013), Bachelier et al. (2006), Satoh and Sugimoto (2006), Sun et al. (2001)).

In the paper the problem is adapted to a stabilization problem by assigning the switching system modes with a partly common left eigenstructure through a state feedback. Utilizing algebraic methods (Datta (2004)), this exposition is generalized to handle one common left eigenvector of the switching multi-input (MI) modes. Preferring the method given in Kouhi and Bajcinca (2011a), the degree of freedom in design is used to construct the modes of the Hurwitz type. The obtained set of LMIs, giving design conditions for state-dependent switching control design, provides a method available in stabilization of the modes.

The outline of this paper is as follows: Following introduction in Sec. 1, Sec. 2 contains some preliminaries related to eigenstructure assignment, Sec. 3 presents the control design conditions, and in Sec. 4 these are reformulated and proven to be applied in state-dependent switched system modes stabilization. Conforming the results, the final two sections follow with an example and concluding remarks.

Throughout the paper, the next notations are used: $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denotes the transpose of the vector $\boldsymbol{x}$ and matrix $\boldsymbol{X}$, respectively, $diag[\,\cdot\,]$ marks a diagonal matrix, for a square matrix $\boldsymbol{X} < 0$ means that $\boldsymbol{X}$ is a symmetric negative definite matrix, the symbol $\boldsymbol{I}_n$ indicates the n-th order unit matrix, $\mathbb{R}$ qualifies the set of real numbers and $\mathbb{R}^{n\times r}$ refers to the set of $n \times r$ real matrices.

⋆ The work presented in this paper was supported by VEGA, the Grant Agency of the Ministry of Education and the Academy of Science of Slovak Republic, under Grant No. 1/0348/14. This support is very gratefully acknowledged.

10.1016/j.ifacol.2015.09.212

2. BASIC PRELIMINARIES

In order to solve the problem of in part common left eigenstructure assignment, the next inequalities and basic existence conditions are adduced.

Proposition 1. (Petersen (1987)) If $\boldsymbol{U}, \boldsymbol{V}$ are real matrices and $\boldsymbol{H}$ is a symmetric positive definite matrix of appropriate dimension, then

$$\boldsymbol{U}^T\boldsymbol{V} + \boldsymbol{V}^T\boldsymbol{U} \leq \boldsymbol{U}^T\boldsymbol{H}\boldsymbol{U} + \boldsymbol{V}^T\boldsymbol{H}^{-1}\boldsymbol{V} \tag{1}$$

Proposition 2. If $\boldsymbol{Q}$ is a real square matrix and $\boldsymbol{E}$ is a singular matrix of appropriate dimension such that

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{E}^T\boldsymbol{Q} = \boldsymbol{Q}^T\boldsymbol{E} \tag{2}$$

then $\boldsymbol{Q}$ takes the structure

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_1 & \boldsymbol{0} \\ \boldsymbol{Q}_3 & \boldsymbol{Q}_4 \end{bmatrix}, \quad \boldsymbol{Q}_1 = \boldsymbol{Q}_1^T \tag{3}$$

Lemma 1. If $\boldsymbol{X}$, $\boldsymbol{Y}$ are real matrices of appropriate dimensions, then

$$\boldsymbol{X}^T\boldsymbol{Y}\boldsymbol{Y}^T\boldsymbol{X} \geq \boldsymbol{X}^T\boldsymbol{Y} + \boldsymbol{Y}^T\boldsymbol{X} - \boldsymbol{I} \tag{4}$$

Proof: Since

$$\big(\boldsymbol{I} - \boldsymbol{Y}^T\boldsymbol{X}\big)^T\big(\boldsymbol{I} - \boldsymbol{Y}^T\boldsymbol{X}\big) \geq 0 \tag{5}$$

$$\boldsymbol{I} + \boldsymbol{X}^T\boldsymbol{Y}\boldsymbol{Y}^T\boldsymbol{X} - \boldsymbol{X}^T\boldsymbol{Y} - \boldsymbol{Y}^T\boldsymbol{X} \geq 0 \tag{6}$$

it is obvious that (6) implies (4). ■

Definition 1. (Horn and Johnson (1995)) Let $\boldsymbol{X} \in I\!R^{n \times n}$ is a square matrix. If a scalar $\lambda \in I\!R$ and a nonzero vector $\boldsymbol{n} \in I\!R^n$ satisfy the equation

$$\boldsymbol{X}\boldsymbol{n} = \lambda\boldsymbol{n} \tag{7}$$

λ is an eigenvalue of $\boldsymbol{X}$, $\boldsymbol{n}$ is a (right) eigenvector of $\boldsymbol{X}$ associated with λ and the pair $(\lambda, \boldsymbol{n})$ is an eigenpair for $\boldsymbol{X}$.

If $\boldsymbol{X}$ has n distinct eigenvalues and n linearly independent associated right eigenvectors, the complete eigenstructure of $\boldsymbol{X}$ is specified by the equations

$$\boldsymbol{M}^T\boldsymbol{X} = \boldsymbol{S}\boldsymbol{M}^T, \qquad \boldsymbol{X}\boldsymbol{N} = \boldsymbol{N}\boldsymbol{S} \tag{8}$$

where

$$\boldsymbol{M}^T = \begin{bmatrix} \boldsymbol{m}_1^T \\ \boldsymbol{m}_2^T \\ \vdots \\ \boldsymbol{m}_n^T \end{bmatrix}, \quad \begin{matrix} \boldsymbol{N} = [\, \boldsymbol{n}_1 \ \boldsymbol{n}_2 \ \cdots \ \boldsymbol{n}_n \,], \\ \boldsymbol{S} = diag\,[\, s_1 \ s_2 \ \cdots \ s_n \,] \end{matrix} \tag{9}$$

$\{s_i, i = 1, 2, \ldots, n\}$ is the set of eigenvalues of $\boldsymbol{X}$ and $\{\boldsymbol{m}_i^T, \ i = 1, 2, \ldots, n\}$, $\{\boldsymbol{n}_i, i = 1, 2, \ldots, n\}$ are the sets of associated left and right eigenvectors, respectively. □

Considering a controllable time-invariant linear system of the form

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{10}$$

where $\boldsymbol{q}(t) \in I\!R^n$, $\boldsymbol{u}(t) \in I\!R^r$ is the state and input vector, respectively, $\boldsymbol{A} \in I\!R^{n \times n}$, $\boldsymbol{B} \in I\!R^{n \times r}$, $rank\boldsymbol{B} = r$ and the control law

$$\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{q}(t) \tag{11}$$

then

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}_c\boldsymbol{q}(t) \tag{12}$$

where

$$\boldsymbol{A}_c = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K} \tag{13}$$

The following lemma is proven.

Lemma 2. For any set of distinct negative real values $\{s_i < 0, \ i = 1, 2, \ldots, r\}$ there exist a constant matrix

$$\boldsymbol{K} = (\boldsymbol{M}_1^T\boldsymbol{B})^{-1}\big(\boldsymbol{M}_1^T\boldsymbol{A} - \boldsymbol{S}_1\boldsymbol{M}_1^T\big) \tag{14}$$

$\boldsymbol{K} \in I\!R^{r \times n}$, such that

$$\boldsymbol{A}_c = \boldsymbol{B}(\boldsymbol{M}_1^T\boldsymbol{B})^{-1}\boldsymbol{S}_1\boldsymbol{M}_1^T + \big(\boldsymbol{I}_n - \boldsymbol{B}(\boldsymbol{M}_1^T\boldsymbol{B})^{-1}\boldsymbol{M}_1^T\big)\boldsymbol{A} \tag{15}$$

The elements of the matrices

$$\boldsymbol{S}_1 = diag\,[\, s_1 \ s_2 \ \cdots \ s_r \,], \quad \boldsymbol{M}_1^T = \begin{bmatrix} \boldsymbol{m}_1^T \\ \boldsymbol{m}_2^T \\ \vdots \\ \boldsymbol{m}_r^T \end{bmatrix} \tag{16}$$

form the stable subset of eigenvalues of the matrix $\boldsymbol{A}_c$ and the associated left eigenvectors, respectively.

Proof: Let $\boldsymbol{m}_i^T$ is the i-th left eigenvector associated with the real eigenvalue s_i of the closed-loop system matrix (13), i.e.

$$\boldsymbol{m}_i^T\boldsymbol{A}_c = \boldsymbol{m}_i^T(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}) = s_i\boldsymbol{m}_i^T \tag{17}$$

Using (16), then (17) can be written in the form

$$\boldsymbol{M}_1^T(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}) = \boldsymbol{S}_1\boldsymbol{M}_1^T \tag{18}$$

which implies (14). Substituting (14) in (13), it is

$$\boldsymbol{A}_c = \boldsymbol{A} - \boldsymbol{A}_s \tag{19}$$

$$\boldsymbol{A}_s = \boldsymbol{B}(\boldsymbol{M}_1^T\boldsymbol{B})^{-1}\boldsymbol{L}, \quad \boldsymbol{L} = \boldsymbol{M}_1^T\boldsymbol{A} - \boldsymbol{S}_1\boldsymbol{M}_1^T \tag{20}$$

and, by straightforward calculation, (19) implies (15). ■

In this eigenstructure assignment problem, a feedback gain matrix $\boldsymbol{K}$ has to be designed so that the closed-loop system has a prescribed subset of eigenvalue spectrum $\sigma_1(\boldsymbol{A}_c) = \{s_h : s_h < 0, \ h = 1, 2, \ldots r\}$ and the rest subset of the eigenvalue spectrum $\sigma_2(\boldsymbol{A}_c) = \{s_h : \mathfrak{Re}(s_h) < 0, \ h = r+1, r+2, \ldots n\}$ has to be stable. Note, while the subset $\sigma_1(\boldsymbol{A}_c)$ is real, the subset $\sigma_2(\boldsymbol{A}_c)$ is generally closed under complex conjugation.

The structure of $\boldsymbol{A}_c$ written in the form (15) comprises two singular matrices and, if a solution exists, the eigenvalue spectra of these singular matrices are (Filasová et al. (2014))

$$\sigma(\boldsymbol{B}(\boldsymbol{M}_1^T\boldsymbol{B})^{-1}\boldsymbol{S}_1\boldsymbol{M}_1^T) = \{s_1, \cdots, s_r, 0, \cdots, 0\} \tag{21}$$

$$\sigma((\boldsymbol{I}_n - \boldsymbol{B}(\boldsymbol{M}_1^T\boldsymbol{B})^{-1}\boldsymbol{M}_1^T)\boldsymbol{A}) = \{s_{r+1}, \cdots, s_n, 0, \cdots, 0\} \tag{22}$$

Because a prescribed stable $\boldsymbol{S}_1$ implies the stable subset $\{s_1, \cdots, s_r\}$ of $\sigma(\boldsymbol{B}(\boldsymbol{M}_1^T\boldsymbol{B})^{-1}\boldsymbol{S}_1\boldsymbol{M}_1^T)$, then the eigenvalue spectrum of the matrix $\boldsymbol{A}_c$, i.e.,

$$\sigma(\boldsymbol{A}_c) = \{s_1, s_2, \cdots, s_n\} \tag{23}$$

be stable if the obtained eigenvalue subset $\{s_{r+1}, \cdots, s_n\}$ of $\sigma((\boldsymbol{I}_n - \boldsymbol{B}(\boldsymbol{M}_1^T\boldsymbol{B})^{-1}\boldsymbol{M}_1^T)\boldsymbol{A})$ be stable.