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A New Anisotropy-Based Control Design Approach for Descriptor Systems Using Convex Optimization Techniques ⋆

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Abstract: In this paper, suboptimal anisotropy-based control problem for linear discrete-time descriptor systems is solved. The obtained conditions are given in terms of LMIs. Numerical example is given.

Keywords: Descriptor systems, anisotropy-based control theory, suboptimal control, LMI, colored noise.

1. INTRODUCTION

While constructing mathematical models of the systems in physical variables, one can get models, which contain both differential (or difference) and algebraic equations. Such models are called descriptor (or singular), and they found wide application in different fields of science and engineering Dai (1989); Stykel (2002).

The descriptor representation is more powerful than the conventional state-space form, but analysis and design methods for descriptor systems are quite different from the classical ones, sometimes they are difficult to be implemented. It is not trivial to extend the methods of normal systems analysis and design on a class of descriptor systems because of the presence of algebraic equations. Algebraic constraints provide the system with some new properties, such as impossibility to solve the system in regard to the derivative, necessity to have sufficiently smooth input signals, and noncausal behavior in discrete-time case (impulse behavior in continuous time).

Some problems, solved for normal systems, are still actual for descriptor systems. One of such problems is a developing of computationally efficient methods of analysis and control design for descriptor systems. This paper is devoted to one of such problems — suboptimal anisotropy-based controller design using convex optimization.

Anisotropy-based control theory originates from Vladimirov (1995, 1996). Information-theoretic representation of random signals lies in the basis of this approach. Anisotropy-based control theory considers the system's reaction on the influence of "colored" noises. "Spectral color" means Kullback-Leibler information divergence from the Gaussian white noise sequence. In this case, the quality criterion is anisotropic norm of the system. This norm lies between normalized $\mathcal{H}_2$-norm and $\mathcal{H}_\infty$-norm of the system. Anisotropy-based analysis problem for normal systems using convex optimization was solved in Tchaikovsky (2011). This result was extended on descriptor systems in Belov (2013). Generalized Riccati inequalities approach to suboptimal anisotropy-based control design was described in Andrianova (2014). But in the listed results inequalities are not strict. As the matrices in constraints are singular, the obtained inequalities are not convex. In Feng (2013), computationally efficient algorithm of suboptimal $\mathcal{H}_\infty$-control design was proposed. This paper extends this algorithm on anisotropy-based case, a novel anisotropy-based bounded real lemma in terms of LMIs (linear matrix inequalities) is formulated and proved. It allows to develop methods of anisotropy-based analysis for descriptor systems.

The paper is organized as follows. In the section 2, basics of anisotropy-based analysis and descriptor systems theory are given. The conditions of a novel bounded real lemma in terms of LMIs for normal and descriptor systems are obtained in the section 3. Suboptimal anisotropy-based controller design problem is solved, based on the novel bounded real lemma, and numerical example is given in the section 4.

2. BACKGORUND

2.1 Descriptor systems

The state-space representation of discrete-time descriptor systems is

$$Ex(k+1) = Ax(k) + Bf(k), \qquad (1)$$
$$y(k) = Cx(k) + Df(k)$$

where $x(k) \in \mathbb{R}^n$ is the state, $f(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^p$ are the input and output signals, respectively, A, B, C and D

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are constant real matrices of appropriate dimensions. The matrix $E \in \mathbb{R}^{n\times n}$ is singular, $\operatorname{rank}(E) = r < n$.

Definition 1. The system (1) is called regular if $\exists\lambda \neq 0$: $\det(\lambda E - A) \neq 0$.

Regularity stands for the existence and uniqueness of the solution for the consistent initial conditions Stykel (2002).

Hereinafter, we suppose that the considered systems are regular. Now we give some definitions, necessary for further presentation.

Definition 2. The transfer function of the system (1) is defined by the expression

$$P(z) = C(zE - A)^{-1}B + D,\ z \in \mathbb{C}. \tag{2}$$

$\mathcal{H}_2$- and $\mathcal{H}_\infty$-norms of the transfer function $P(z)$ are defined as follows

$$\|P\|_2 = \left(\frac{1}{2\pi}\int_0^{2\pi} \operatorname{tr}\left(P^*(e^{i\omega})P(e^{i\omega})\right)d\omega\right)^{\frac{1}{2}},$$

$$\|P\|_\infty = \sup_{\omega\in[0,2\pi]} \sigma_{max}\left(P(e^{i\omega})\right)$$

where $\sigma_{max}\left(P(e^{i\omega})\right)$ is the maximum singular value of the transfer function $P(z)$.

Definition 3. The system (1) is called admissible if it is regular, causal ($\deg\det(zE - A) = \operatorname{rank} E$), and stable ($\rho(E, A) = \max|\lambda|_{\lambda\in\{z|\det(zE-A)=0\}} < 1$). For more information, see Dai (1989); Xu (2006).

For the regular system (1) there exist two nonsingular matrices Dai (1989) $\overline{W}$ and $\overline{V}$ such that $\overline{W}E\overline{V} = \operatorname{diag}(I_r, 0)$.

Consider the following change of variables

$$\overline{V}^{-1}x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{3}$$

where $x_1(k) \in \mathbb{R}^r$ and $x_2(k) \in \mathbb{R}^{n-r}$.

By left multiplying the system (1) on the matrix $\overline{W}$ and using the change of variables (3), one can rewrite the system (1) in the form Dai (1989)

$$\begin{aligned} x_1(k+1) &= A_{11}x_1(k) + A_{12}x_2(k) + B_1 f(k), \\ 0 &= A_{21}x_1(k) + A_{22}x_2(k) + B_2 f(k), \\ y(k) &= C_1x_1(k) + C_2x_2(k) + Df(k) \end{aligned} \tag{4}$$

where

$$\overline{W}A\overline{V} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\quad \overline{W}B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$
$$C\overline{V} = [\,C_1\ C_2\,]. \tag{5}$$

Matrices $\overline{W}$ and $\overline{V}$ are found from the singular value decomposition (SVD)

$$E = U\operatorname{diag}(S, 0)H^{\mathrm{T}}.$$

Here U and H are real ortogonal matrices, S is a diagonal $r\times r$-matrix, it is formed by nonzero singular values of the matrix E

$$\overline{W} = \operatorname{diag}(S^{-1}, I_{n-r})U^{\mathrm{T}}, \qquad \overline{V} = H.$$

Representation (4) is called SVD canonical form Dai (1989). Note that the system is causal if $\det(A_{22}) \neq 0$, and stable if $\rho(A_{11} - A_{12}A_{22}^{-1}A_{21}) < 1$ Xu (2006).

While solving control problems for descriptor systems it is necessary not only to provide stability of dynamical subsystem, but also to avoid undesirable noncausal behavior. So, for descriptor systems there exist such concepts as causal controllability and stabilizability. Discuss them in detail. Consider a state feedback control in the following form:

$$f(k) = F_c x(k) + h(k) \tag{6}$$

where $F_c \in \mathbb{R}^{m\times n}$ is a constant real matrix, $h(k)$ is a new input signal. The closed-loop system may be written in the form

$$Ex(k+1) = (A + BF_c)x(k) + Bh(k). \tag{7}$$

Definition 4. The system (1) is called causal controllable if there exists a state feedback control in the form (6) such that the closed-loop system (7) is causal.

Causal controllability can be easily checked by the following rank condition Dai (1989).

Theorem 5. The system (1) is causal controllable if

$$\operatorname{rank}\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} = \operatorname{rank}(E) + n.$$

Stabilizability of descriptor systems is characterized by ability to control nonstable modes of the dynamical subsystem.

Definition 6. The system (1) is called stabilizable if there exists a state feedback control in the form $f(k) = F_{st}x(k)$ such that the pair $(E, A + BF_{st})$ is stable.

2.2 Mean anisotropy of the sequence and anisotropic norm of the system

Let $W = \{w(k)\}_{k\in\mathbb{Z}}$ be a stationary sequence of square-integrable random m-dimensional vectors. The sequence W can be generated from the Gaussian white noise sequence V with zero mean and identity covariance matrix by an admissible shaping filter with a transfer function $G(z) = C_G(zE_G - A_G)^{-1}B_G + D_G$. Mean anisotropy of the signal is Kullback-Leibler information divergence from probability density function (p.d.f.) of the signal to p.d.f. of the Gaussian white noise sequence.

Mean anisotropy of the sequence may be defined by the filter's parameters, using the expression

$$\overline{\mathbf{A}}(W) = -\frac{1}{4\pi}\int_{-\pi}^{\pi} \ln\det\frac{mS(\omega)}{\|G\|_2^2}d\omega$$

where $S(\omega) = \widehat{G}(\omega)\widehat{G}^*(\omega)$, $(-\pi \leqslant \omega \leqslant \pi)$, $\widehat{G}(\omega) = \lim_{l\to 1} G(le^{i\omega})$ is a boundary value of the transfer function $G(z)$.

Remark 1. Mean anisotropy of the random sequence W, generated by shaping filter $G(z)$, is fully defined by its parameters, so the notations $\overline{\mathbf{A}}(G)$ and $\overline{\mathbf{A}}(W)$ are equivalent.

Mean anisotropy of the signal characterizes its "spectral color", i.e. the difference between the signal and the Gaussian white noise sequence. If $\overline{\mathbf{A}}(W) = 0$, then the signal is the Gaussian white noise sequence. If $\overline{\mathbf{A}}(W) \to \infty$, the signal is a determinate sequence. For more information, see Vladimirov (2006, 1995).

Let $Y = PW$ be an output of the linear discrete-time descriptor system $P \in \mathcal{H}_\infty{}^{p\times m}$ with a transfer function