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On the geodetic assumptions for the measurement of the neutrino velocity

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ABSTRACT

The geodetic assumptions for the measurement of the neutrino velocity by applying the standard time of the Global Positioning System (GPS) and the length of the basis, which is defined by the geodetic measurements, are presented. The experimental data were taken from the OPERA and ICARUS experiments where neutrino emission from CERN LHC accelerator to the Gran Sasso detector had been investigated. A distance between accelerator and detector is about 730 km. A time interval of the neutrinos travelling was measured by the benchmark clocks, which were calibrated according to the standard GPS time signals received from the GPS satellites. The accuracy of the characteristics of the GPS time signals and time intervals of the GPS signals travels between the satellites and ground-based receivers are considered in the calculations of the neutrino velocity. An influence of the correlation between time intervals, and the correlation between the speed of light and the time intervals is estimated. The measurement conditions, under which it is possible to measure the neutrino velocity with adequate precision to the measurements of the speed of light, and which are related to the necessary accuracies of the basis' length and time intervals are given and discussed. The geodetic presumptions for the measurement of the neutrino velocities during *Project X* experiment and for the usage of the basis Earth–Moon are proposed.

1. Introduction

The challenge to measure the neutrino velocity with the same or, at least, near accuracy as the speed of light still remains. Some efforts were done in the frames of the OPERA, ICARUS and MINOS experiments [1–9]. The geodetic methods play an important role in the measurement of the neutrino velocity by defining the length of the basis and in providing the time scale. Some improvements to measure the length of the basis were done by geodesists in ICARUS experiment [5]. Therefore, the accuracy of the neutrino velocity is still of two orders less than the accuracy of the speed of light [10–14].

The calculations of the neutrino velocity taking into account the accuracy characteristics of the GPS time signals and time intervals of the GPS signals travels between the satellites and ground-based receivers are presented. The GPS time moments received by the ground-based receivers are correlated. As a result, an influence of the correlation between time intervals and the correlation between the speed of the light and the time intervals was estimated. The measurement conditions, under which it is possible to measure the neutrino velocity with adequate precision, and which are related to the necessary accuracies of the basis' length and time intervals are given and discussed.

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2. Theoretical assumptions

The analysis of the accuracy of the neutrino velocity's measurement including the influence of the correlation of the corresponding parameters of the composed speed function on the estimation of the standard deviation of this function is given. The covariance $K(t_{cer}, t_{sas})$ between two moments in time, t_{cer} and t_{sas}, which are recorded at CERN emitter and the Gran Sasso detector, can be calculated accordingly:

$$K(t_{cer}, t_{sas}) = M\{(t_{cer} - Mt_{cer})(t_{sas} - Mt_{sas})\} = M\{\delta t_{cer} \cdot \delta t_{sas}\}, \quad (1)$$

here M – the symbol of a mean; $\delta t_{cer}, \delta t_{sas}$ – the random errors of the corresponding moments in time. The moments in time are received from the same GPS satellites, therefore, the systematic errors of the moments in time are equal and the influence of these errors on the differences between the moments in time is negligible.

Further:

$$t_{cer} = t_{gps} + \tau_{cer} \quad (2)$$

and

$$t_{sas} = t_{gps} + \tau_{sas} = t_{gps} + (\tau_{cer} + \tau_m), \quad (3)$$

here $\tau_{cer}, \tau_{sas} = \tau_{cer} + \tau_m$ – corresponding time intervals of the travelling time of the signals from the GPS satellites to the ground-based receivers, τ_m – the alteration of the time interval caused by the different distances between the satellites and CERN, and between the satellites and the Gran Sasso ($S_{gps-cern} \neq S_{gps-sasso}, \quad \tau_{gps-cern} \neq \tau_{gps-sasso}$).

Therefore, the formula (1) assumes such form:

$$K(t_{cer}, t_{sas}) = M\{(\delta t_{gps} + \delta\tau_{cer})(\delta t_{gps} + \delta\tau_{sas})\} = M(\delta t_{gps})^2 = \sigma^2_{t_{gps}}, \quad (4)$$

here $\sigma_{t_{gps}}, \sigma_{\tau_{cer}}$ – the standard deviations of the corresponding GPS moment in time and time interval. The means of multiplication of the random errors of the mixed quantities (as independent) are equal to zero, thus $M(\delta t_{gps} \cdot \delta\tau_{cer}) = M\delta t_{gps} \cdot M\delta\tau_{cer} = 0$, $M(\delta t_{gps} \cdot \delta\tau_{sas}) = 0$, $M(\delta\tau_{cer} \cdot \delta\tau_{sas}) = 0$, $M\delta\tau_{gps} = M\delta\tau_{cer} = M\delta\tau_{sas} = 0$.

The time interval τ_n of the neutrino travelling from the CERN emitter to the Gran Sasso detector is determined by the formula:

$$\tau_n = t_{sas} - t_{cer}. \quad (5)$$

The standard deviations of the corresponding moments in time are:

$$\sigma^2_{t_{cer}} = \sigma^2_{t_{gps}} + \sigma^2_{\tau_{cer}}, \quad (6)$$

$$\sigma^2_{t_{sas}} = \sigma^2_{t_{gps}} + \sigma^2_{\tau_{sas}}, \quad (7)$$

The standard deviation σ_{τ_n} of the time interval τ_n of the neutrino travelling can be calculated by applying Eqs. (4)–(7) to the formula

$$\sigma^2_{\tau_n} = \sigma^2_{t_{sas}} + \sigma^2_{t_{cer}} + 2K(t_{sas}, t_{cer})\left(\frac{\partial\tau_n}{\partial t_{sas}}\right)_0\left(\frac{\partial\tau_n}{\partial t_{cer}}\right)_0 = 2\sigma^2_{t_{gps}} + \sigma^2_{\tau_{cer}} + \sigma^2_{\tau_{sas}} - 2\sigma^2_{t_{gps}} = \sigma^2_{\tau_{cer}} + \sigma^2_{\tau_{sas}} = 2\sigma^2_{\tau_{cer}}, \quad (8)$$

here $\sigma_{\tau_{cer}} = \sigma_{\tau_{sas}}$ is accepted.

The ratio of the moments in time of the GPS satellite's time generators is $\sigma_{t_{gps}}/t_{24h} = 10^{-14}$, so $\sigma_{t_{gps}} = 0.86$ ns.

The calculations of, for example, the accuracy of the measurement of the neutrino speed c_n in accordance with the results of the OPERA's experiment [1] can be written

$$c_n = S \cdot \tau_n^{-1} = S(\tau_e - \delta_n)^{-1} = S\tau_c^{-1}(1 - \delta_n\tau_c^{-1})^{-1} = c(1 + \delta_n\tau_c^{-1} + \delta_n^2\tau_c^{-2} + \ldots), \quad (9)$$

here $S \approx 730$ km – the length of the basis CERN – Gran Sasso, $\tau_n = \tau_c - \delta_n$, $\tau_c = S \cdot c^{-1}$ – a time interval of the light travelling between CERN – Gran Sasso, $\delta_n \to 60$ ns – the value of the time interval of the neutrino travelling which was less than the time interval of the light travelling (systematic trend), c – speed of light. First order members of Eq. (9) are applied, because the influence of the second order members is at a level about 10^{-2} m/s. The results of the calculation:

$$c_n = c(1 + 24.7 \cdot 10^{-6} + 6.12 \cdot 10^{-10}) = c + \delta c_n. \quad (10)$$

A correction of the neutrino travelling speed δc_n, which shows by what value the neutrino travelling speed is higher than the speed of light, is equal to

$$\delta c_n = 24.7 \cdot 10^{-6}c + 6.12 \cdot 10^{-10}c = 7404.9 \text{ m/s}. \quad (11)$$

Therefore, according to the calculations, the neutrino travelling speed is

$$c_n = c + \delta c_n = 299792458 + 7405 = 299799863 \text{ m/s}. \quad (12)$$

In order to calculate the standard deviation of the neutrino travelling speed c_n, two linear members of the formula (9) are used. It can be written

$$\sigma^2_{c_n} = (1 + \delta_n\tau_c^{-1})^2\sigma_c^2 + c^2(\delta_n^2\tau_c^{-4} \cdot \sigma^2_{\tau_c} + \tau_c^{-2}\sigma^2_{\delta_n}) + 2K(c, \tau_c)\left(\frac{\partial c_n}{\partial c}\right)_0\left(\frac{\partial c_n}{\partial(1 + \delta_n\tau_c^{-1})}\frac{\partial(1 + \delta\tau_c^{-1})}{\partial\tau_c}\right)_0, \quad (13)$$

A covariance matrix $K(c, \tau_c)$, when the second and higher order members are ignored, is

$$\begin{aligned} K(c, \tau_c) &= M\{(c - Mc)(\tau_c - M\tau_c)\} \\ &= M\{\delta c \cdot [S \cdot c^{-1} - M(Sc^{-1})]\} \\ &= M\left\{\delta c\left[S \cdot M(c^{-1}) + \frac{\partial\tau_c}{\partial c}(c - Mc) - S \cdot M(c^{-1})\right]\right\} \\ &= M\{-\delta c \cdot S \cdot c^{-2} \cdot \delta c\} = -Sc^{-2}M(\delta c)^2 \\ &= -Sc^{-2}\sigma_c^2 = -\tau_c c^{-1}\sigma_c^2, \end{aligned} \quad (14)$$

here $\delta c = c - Mc$, $\tau_c = S/c$, $M\tau_c = S/Mc$.

What is more, by using (13) and (14), it can be written

$$\sigma^2_{c_n} = (1 + \delta_n\tau_c^{-1})^2\sigma_c^2 + c^2(\delta_n^2\tau_c^{-4} \cdot \sigma^2_{\tau_c} + \tau_c^{-2}\sigma^2_{\delta_n}) + 2(-\tau_c c^{-1}\sigma_c^2)(1 + \delta_n\tau_c^{-1})c\delta_n(-\tau_c^{-2})$$

and

$$\begin{aligned} \sigma^2_{c_n} &= (1 + \delta_n\tau_c^{-1})(1 + 3\delta_n\tau_c^{-1})\sigma_c^2 + c^2(\delta_n^2\tau_c^{-4} \cdot \sigma^2_{\tau_c} + \tau_c^{-2}\sigma^2_{\delta_n}) \\ &= A\sigma_c^2 + c^2\sigma_e^2 = \sigma^2_{c_{n1}} + \sigma^2_{c_{n2}}, \end{aligned} \quad (15)$$