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IFAC-PapersOnLine 48-11 (2015) 384–389

Automatic Synthesis of Control for Multi-Agent Systems with Dynamic Constraints

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Abstract: The paper solves the problem of automatic synthesis of a control system. The result of the control synthesis is a synthesizing control function that depends on the current state of the object and provides the optimal control with the best functional value from any initial point of some initial domain. A network operator method is used to solve the problem. We apply the proposed approach for control synthesis of a robotic team. The main challenge with such multi-agent mobile systems is to provide control meeting inter agent dynamic constraints**.** The method's performance is demonstrated through simulations on a team of two mobile robots performing a parking task.

Keywords: control synthesis, multi object control, dynamic constraints, symbolic regression

1. INTRODUCTION

The synthesis of a control system seems to be a paramount and at the same time a highly challenging problem of the modern science. Solution of the synthesis problem provides an opportunity to define any time the control value basing on the object's state in order to reach the control goals. Moreover, synthesized control function has to be constructed in such a way to solve the problem from not only one initial state but from any initial state from a specified domain.

Analytical solutions apply to small class of the systems. That is especially true for the non-linear systems subclass. The control function to be found and its derivatives can have discontinuities.

Currently the control synthesis problem is solved in most cases manually basing on the developer's experience and application of some semi-analytical methods like analytical construction of optimal regulators [Letov, A.M., 1960, Lee, E.B., Marcus, L., 1986].

The application of modern computational methods of symbolic regression like genetic programming [Koza, J.R., 1992,], grammatical evolution [O'Neill, M., Ryan, C., 2002] and analytical programming [Zelinka, I., 2002] make it possible to automate the synthesis process. They search the space of mathematical expressions to find the control function that best fits given criteria. They do allow to organize functional search but still rarely used for control synthesis because of the complexity of the problem. So we met just a few papers devoted to the subject such as control of a reverse pendulum [Koza, J.R., *et al.*, 1999] or a flight control system [Bourmistrova, A., Khantsis, S., 2010].

In the present paper we use another method of symbolic regression - the network operator method [Diveev, A.I., 2012, Diveev, A.I., *et al.*, 2013] that had been reported to successfully synthesize control systems for various problems. We apply the network operator method to solve the problem of the control system synthesis of multiple autonomous mobile robots.

Formations of multi-agent systems, such as mobile robots, require individual agents to satisfy their kinematic equations while constantly maintaining inter agent dynamic constraints.

All applications of mobile robotic team address the issue of path planning and motion coordination [Parker, L.E., 2009]. This task has been extensively studied since the 1980s. While many techniques have been developed to address this challenge, however, finding the optimal solution to path planning in multi-robot system is NP-hard [Stephen, O., *et al.*, 2012].

This paper describes our computational approach that allow to provide advance path control for each robot while coordinating the trajectories to avoid collisions with each other in order to reach robots' objectives such as parking in the specified position.

To provide proper cooperation among robots, we consider a team of robots as a single system with extended state vectors and control vectors. The control should satisfy the dynamic constraints, which prevent collisions between objects regarding its dimensions. We made simulations for a team of two mobile robots performing a parking task. The method allowed us to find the synthesizing function that provided the goal achievement and met constraints requirements for each robot in the team with a good value of the quality criterion.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.09.216

2. PROBLEM OF CONTROL SYNTHESIS

In technical terms the synthesis problem involves the construction of such a control module that produces control basing on the received data about object's state and this control makes an object to achieve the goal with optimal values of some given functional. The control module must produce an optimal control whatever is the object's state, not only one preliminary prescribed. So, solving the traditional problem of optimal control for a plant in some known initial state and receiving an optimal control program as a function of time could not meet the requirements of the synthesis problem. The control module must produce an optimal control for different possible initial states of the plant as well as for any possible states the plant can occur during the control process even if they does not belong to the optimal trajectory.

Conventionally the plant is described by the system of ODEs. Each differential equation describes variations of one coordinate of state space in time. The right parts of the system contain some free parameters called a control vector. Our goal is to find a function that describes how the components of the control vector depend on the components of the vector of states. If we insert the received function into the right parts of the differential equations, we will get the system of differential equations without free parameters. And any particular solution of the received system of differential equations for every initial state of the plant (from a given space) will be an optimal trajectory by some given criteria. The same optimal trajectory we can receive by solving the problem of optimal control with the same criteria and for only one initial state of the plant. But in this case the received optimal control is a function of time and inserting it into the right parts of the differential equations gives a non-autonomous system of equations.

The distinguishing feature of the synthesis problem is that a control is searched as a function of state coordinates so we can choose any initial conditions from some domain and receive new optimal trajectories which are the same as those trajectories obtained from solving several times the problem of optimal control with the chosen initial conditions. Therefore solving the synthesis problem is equal to solving a multitude of the problems of optimal control.

Let us assume that for any initial conditions from some area of state space it is possible to solve the problem of optimal control. Hence any value of the state vector corresponds to some optimal value of control vector, in other words there is a function that describes how the optimal control depends on the values of state vector. This means that there is a solution of the synthesis problem for some area of state space.

3. CONTROL SYNTHESIS OF MULTI-AGENT NONLINEAR SYSTEMS

Let the models of N controlled objects be

$$\dot{\mathbf{x}}^i = \mathbf{f}^i\left(\mathbf{x}^i, \mathbf{u}^i\right),\ i = \overline{1, N}, \qquad (1)$$

where $\mathbf{x}^i$ is a state vector of the object i, $\mathbf{u}^i$ is a control vector of the object i, $\mathbf{x}^i \in \mathrm{R}^{n_i}$, $\mathbf{u}^i \in \mathrm{U}_i \subseteq \mathrm{R}^{m_i}$, $m_i \le n_i$, U_i is a compact set in the corresponding vector space R^{m_i}, $i = \overline{1, N}$.

Let us define sets of initial conditions

$$\mathbf{x}^i(0) \in \mathrm{X}_0^i \subseteq \mathrm{R}^{n_i},\ i = \overline{1, N}, \qquad (2)$$

and terminal conditions

$$\varphi_{j,i}\left(\mathbf{x}^i\left(t_f^i\right)\right) = 0,\ j = \overline{1, r_i},\ i = \overline{1, N}, \qquad (3)$$

where $t_f^i \le t^+$, t^+ is given and defines an upper bound of the acceptable time of control.

Positions of the space restriction points of the objects are described by the following kinematic equations

$$\mathbf{y}^i = \mathbf{g}^i\left(\mathbf{x}^i\right),\ i = \overline{1, N}, \qquad (4)$$

where $\mathbf{y}^i \in \mathrm{R}^{l_i}$, $i = \overline{1, N}$.

Constraints are given as following

$$b_i\left(\mathbf{y}^i, \mathbf{y}^j\right) < 0,\ i \ne j,\ i, j = \overline{1, N}, \qquad (5)$$

where $b_i\left(\mathbf{y}^i, \mathbf{y}^j\right): \mathrm{R}^{l_i} \times \mathrm{R}^{l_j} \to \mathrm{R}^1$.

The condition (5) means the absence of collisions between objects i and j.

The quality functional are

$$J_i = \int_0^{t_f^i} f_0^i\left(\mathbf{x}^i(t), \mathbf{u}^i(t)\right) dt \to \min,\ i = \overline{1, N}, \qquad (6)$$

where $f_0^i\left(\mathbf{x}^i(t), \mathbf{u}^i(t)\right)$ are continuous functions limited from below, $i = \overline{1, N}$.

It is necessary to find a synthesizing control function in a form

$$\mathbf{u}^i = \mathbf{h}^i\left(\mathbf{x}^1, \ldots, \mathbf{x}^N\right), \qquad (7)$$

where $\mathbf{h}^i\left(\mathbf{x}^1, \ldots, \mathbf{x}^N\right): \mathrm{R}^{n_1} \times \ldots \times \mathrm{R}^{n_N} \to \mathrm{R}^{m_i}$

The synthesizing function (7) has to make the system (1) achieve an objective state

$$\dot{\mathbf{x}}^i = \mathbf{f}^i\left(\mathbf{x}^i, \mathbf{h}^i\left(\mathbf{x}^1, \ldots, \mathbf{x}^N\right)\right),\ i = \overline{1, N}, \qquad (8)$$

$$\forall \mathbf{x}^i(0) \in \mathrm{X}_0^i \subseteq \mathrm{R}^{n_i},\ i = \overline{1, N},$$