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Measurement of distorted exponential signal components using maximum likelihood estimation

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ABSTRACT

Exponential signal is a suitable stimulus for dynamic ADC testing because of the simplicity of the generating RC circuit. A potential distortion source of the ideal exponential shape is the dielectric absorption of the capacitor, whose effect can be represented by additional superimposed exponential components with longer time constant and smaller peak value. Measurement of the distortion of the exponential signal by using a reference waveform recorder with known nonlinearity is the initial step in the calibration of an ADC testing stand with exponential stimulus, along with the assessment of its uncertainty. Lack of the orthogonality of stimulus signal components makes classical analysis methods difficult to apply.

This paper presents a method for measurement of multiexponential signal components as an example of the more general task of signal decomposition where signal components are non-orthogonal. The proper optimization procedure based on the ML method will be presented, which usually reaches the global minimum of the cost function. Effectiveness will be shown by simulation, and by application to measurement of a multiexponential signal acquired by a reference waveform recorder with known error parameters.

1. Introduction

Exponential input stimulus signal is attractive for dynamic ADC tests because of the simplicity of the generating RC circuit [1,2]. The estimation of the testing error requires measurement of the distortion of the stimulus signal. Despite the simplicity of the RC circuit there is a distortion in the exponential signal: distortion components are caused by dielectric imperfections of the capacitors in the discharging circuit. The exponential output voltage is connected without any interfacing as dynamic stimulus signal to the input of ADC under test. Distorting elements in the circuit model of a capacitor, which represent the dielectric absorption, are additional serial R_iC_i (for $i = 2,3\ldots$) branches connected in parallel to the primary capacitor C_1 (Fig. 1). Each parasitic R_iC_i branch generates a superimposed exponential component with longer time constant and smaller peak value than the main capacitor, and also slightly changes the main time constant [3].

In order to determine the testing uncertainty, the distorting exponential components must be measured beforehand by a reference waveform recorder (RWR) with known nonlinearity. For the determination of these parameters from the data record a convenient method for estimation of exponential components is needed. Since the exponential functions are not orthogonal, the task to determine parameters of this function is not easy. For this specific problem there are known some determination methods which work well for good Signal-to-Noise Ratio

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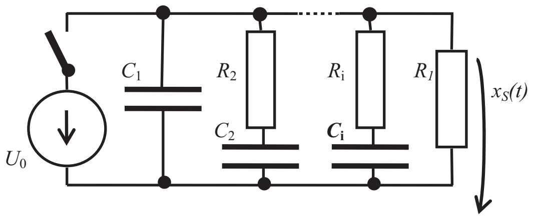

Fig. 1. Circuit model of the exponential signal generator.

(SNR). The most known is Pronýs method [4,10]. In general the number of the superimposed exponential components is unknown. An additional superimposed component is the noise from various interfering sources.

Here the maximum likelihood method is suggested as an alternative to Prony's analytical method which is the straightforward method of identification of the signal components of the stimulus signal acquired by the RWR. Smaller sensitivity to the superimposed noise and the possibility to eliminate the effect of the nonlinearity of the recorder are the main advantages of maximum likelihood method in comparison to Prony's method. The ML method is characterized by larger computational complexity but with the advantage of lower sensitivity to clipping of the signal and to the shape of the quantization noise [6,7,8]. As a parameter estimation method, ML is asymptotically optimal. It also allows to involve the a priori known nonlinearity of RWR in the calculations. A further advantage is the possibility to avoid the convergence into the local minima by appropriate setting of the noise parameters.

The computational complexity is related to the multidimensional optimization of the parameters (A_i, B_i). Anyway, the aim is to achieve best matching of the considered input signal to the true code intervals corresponding to the recorded digital shape. The best matching is assessed by the maximum likelihood procedure, taking into account also the true transfer function of the RWR. The speed of convergence of the parameter estimation also depends on the setting of the initial values at the beginning of the optimization procedure.

Determination of the multiexponential signal components by the ML method will be studied in Section 2. The efficiency of the proposed ML method under various conditions will be studied by simulation, and will be compared to Prony's method in Section 3. Results of the experimental verification and assessment of the estimation uncertainty will be shown in Section 4. A brief summary of the achieved results and further development of the ML method in general signal identification will be provided in Section 5.

2. Mathematical model

As mentioned in [2], dynamic ADC testing using exponential stimulus signal is dominated by the time constant and by the peak value of the basic exponential component. The parameters of basic components are determined by the precise circuit model which generates exponential stimulus, taking into account dielectric absorption. Distorting elements in the circuit model of a capacitor, which represent the dielectric absorption, are the additional serial R_iC_i (for $i = 2, 3\ldots$) circuits, connected in parallel to the primary capacitor C_1 (Fig. 1). Each parasitic R_iC_i circuit generates a superimposed exponential component with longer time constant and smaller peak value than the main capacitor, and also slightly changes the main time constant [3].

The output signal $x_S(t)$ for only one absorption circuit R_2,C_2 in parallel with the primary capacitor C_1 and discharging resistor R_1 can be determined analytically (1) by formula

$$x_s(t) = A_1e^{-B_1t} + A_2e^{-B_2t} \tag{1}$$

The values of time constants and the peak values of two exponential components are:

$$B_{1,2} = \frac{1}{T}(1 \pm a),$$
$$T = \frac{2C_1R_1C_2R_2}{C_1(R_1+R_2)+C_1R_1}, \quad a = \sqrt{1-\frac{4C_1R_1C_2R_2}{[C_1(R_1+R_2)+C_1R_1]^2}}$$
$$A_1 = \frac{U_0}{B_1-B_2}\left(\frac{1}{C_1R_1}-B_2\right); \quad A_2 = U_0 - A_1 \tag{2}$$

The final exponential signal with distorting superimposed components must be known in order to assess the ADC testing stand uncertainty. The method proposed in the paper [4] utilizes Prony's analytical method for the signal acquired by RWR for equidistant sampling. The measurement precision was increased by the measurement of the time instants for known values of DC threshold voltages and by the application of Prony's modified method [9,10]. Drawback of both analytical methods is their high sensitivity on the superimposed noise and their inability to estimate the DC parameter.

2.1. Modeling of the input samples

The exponential signal for ADC testing distorted by the additional exponential components with bigger time constants is represented by

$$x_S(t) = A_1e^{-B_1t} + \sum_{i=2}^{L} A_ie^{-B_it} + C + n(t) \tag{3}$$

where $A_i << A_1$, $B_i << B_1$, $i = 2, 3,\ldots$

While parameters A_1 and B_1 represent the basic exponential signal determined by the primary values R_1, C_1, the parameters A_i and B_i ($i = 2,3,\ldots$) represent the distorting exponential components. Number L is the number of all exponential components assumed in stimulating signal (3). The constant C in (3) describes the offset of the whole exponential signal. The distorted multiexponential signal is corrupted by additional (mainly thermal) noise of the analog components and by interferences from external sources ($n(t)$). The superimposed input noise $n(t)$ is assumed to have Gaussian distribution with zero mean and variance σ_n.

The sampled input signal $x_s(j)$ with sampling period τ_s is represented by (4) before its quantization in the RWR.

$$x_s(j) = \sum_{i=1}^{L} A_i \cdot e^{-B_i j\tau_S} + C + n(j\tau_S) \tag{4}$$