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# The speed bi-gradient method for model reference adaptive control of affine cascade systems*

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**Abstract:** The model reference adaptive control synthesis problem for affine systems consisting of two subsystems is considered. The reference model specifies the desired state of output subsystem. The speed bi-gradient method to design continuous or sliding mode control algorithms with tunable manifold is proposed. The control law ensures both the boundedness of the closed loop system for bounded initial conditions and desired motion of the output subsystem under parameter uncertainty. The conditions of applicability and stability analysis of proposed algorithm are presented.



*Keywords*: Reference adaptive control, Nonlinear control systems, Sliding surfaces, Stability.

## 1. INTRODUCTION

The *speed bi-gradient method* (SBGM) (Myshlyayev, 2002, 2009, 2013) is the novel adaptive control design strategy for nonlinear cascade plants with two subsystems, and with control objective depending on trajectories of output subsystem. SBGM is close to speed-gradient method (Fradkov, 1979), and adaptive control law, merging parameter identification and sliding mode, that was proposed and analytically studied by Fradkov and Andrievsky (1988, 1999). The particular case of SBGM is the Sliding mode with tuning surface (SMTS) method proposed by Myshlyayev (1999, 2013). The main contribution of SMTS compared to the sliding mode control (Utkin, 1977, 2009) is a tuning sliding manifold.

Myshlyayev successfully applied SBGM for solving several adaptive control problems, such as synchronization of two-pendulum system with a drive motor (2013), adaptive control of the electromechanical plant consisting of Lagrangian output subsystem and affine input subsystem (2014a), and adaptive control of a single-axis vibratory gyroscope with a drive motor (2014b).

This paper is focused on the specification of the *speed bi-gradient method* (SBGM) for the control synthesis of affine cascade systems consisting of two subsystems under parameter uncertainty. The idea of SBGM is to change both the tunable parameters and control law along the gradient of the speed of change of the corresponding objective function. The main objective function depends on deviation of output subsystem state trajectory from the reference model state trajectory. The secondary one depends on deviation of system motion from the desired interception of the surfaces. Both the boundedness of the closed loop system for bounded initial conditions and desired motion of the output subsystem are guaranteed. The proposed method can be used to design continuous control algorithms as well as algorithms of SMTS by applying the identical design strategy.

The formulation of nonlinear adaptive control synthesis problem is given in Section 2. In Section 3, SBGM with reference model is described. The design procedure for the affine nonlinear plant with the reference model based on the considered approach is presented in Section 4. The simulation results demonstrate that the control objectives are achieved.

## 2. PROBLEM FORMULATION

Consider cascade plant consisting of output subsystems $S_1$ and input subsystems $S_2$

$$S_1 : \dot{\mathbf{x}}_1 = \mathbf{f}_1(\mathbf{x}_1, \xi) + \mathbf{g}_1(\mathbf{x}_1, \xi)\mathbf{x}_2, \qquad (1)$$
$$S_2 : \dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}, \xi) + \mathbf{g}_2(\mathbf{x}, \xi)\mathbf{u},$$

where $\mathbf{x}^{\mathrm{T}} = (\mathbf{x}_1^{\mathrm{T}}\ \mathbf{x}_2^{\mathrm{T}}) \in R^n, \mathbf{u} \in R^m$ are system state and input vectors respectively; $\mathbf{x}_2 \in R^m$, $\xi \in \Xi$ is a vector of unknown parameters, $\Xi$ is a set of admitted values of $\xi$. Assume $\det \mathbf{g}_2(\cdot) \neq 0$ and $\|\mathbf{g}_2^{-1}(\mathbf{x}_1, \xi)\| \leq C_\xi$ for $\forall \xi \in \Xi, \mathbf{x} \in R^n$.

Introduce the reference model of the output subsystem $S_1$

$$\dot{\mathbf{x}}_1^* = \mathbf{f}_1^*(\mathbf{x}_1^*) + \mathbf{g}_1^*(\mathbf{x}_1^*)\mathbf{r}, \qquad (2)$$

where $\mathbf{x}_1^* \in R^{n-m}$ is a state vector of reference model; $\mathbf{r} \in R^m$ is a reference model input.

***Assumption M.*** The system (4) is input-to-state stable, and $\|\mathbf{r}\| \leq C_r, \|\dot{\mathbf{r}}\| \leq C_{\dot{r}}$.


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It is necessary to find control law $\mathbf{u}$ which guarantees both the boundedness of all trajectories, and the achievement of the objective inequality

$$Q(\mathbf{e}_1) \le \Delta_{e_1} \tag{3}$$

for all $t \ge t_*$ under parametric uncertainties, where $\Delta_{e_1} > 0$, $t_* > 0$, $Q(\mathbf{e}_1)$ is a local objective functional; $\mathbf{e}_1 = \mathbf{x}_1 - \mathbf{x}_1^*$ is a tracking error; $\mathbf{x}_1^*$ is a desired state trajectory of the subsystem $S_1$.

***Assumption Q***. $Q(\mathbf{e}_1)$ is non-negative, uniformly continuous in region $\{(\mathbf{e}_1) : \|\mathbf{e}_1\| \le \beta\}$ and $\inf_{t\ge 0} Q(\mathbf{e}_1) \to +\infty$ as $\|\mathbf{e}_1\| \to \infty$.

In the stability theory, considered problem formulation relates to the partial dissipativity requirement. If $Q(\mathbf{e}_1) \to 0$ as $t \to \infty$, the problem relates to the partial asymptotic stability requirement.

## 3. SYNTHESIS

### *3.1 Technique of synthesis*

The speed bi-gradient approach consists of three stages. At the first stage, the "ideal" virtual control for output subsystem is designed. The "ideal" virtual control ensures the achievement of the control objective (3) for the subsystem $S_1$ of the system (1) assuming that plant's parameters are known. At the second stage, unknown parameters of a virtual control are replaced with tunable ones, and adaptation law is designed to achieve the control objective (3) under parameter uncertainty. At the third stage, deviation from the *interception of the discontinuity surfaces* that is difference between input subsystem's output and tuning virtual control is selected. Control law ensuring system trajectories converge to the manifold is designed. Consider all three stages in detail.

***Stage 1.*** Introduce a virtual control that is the desired input of the output subsystem for subsystem $S_1$ of the system (1) as a vector-function $\mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta}) \in R^{n-m}$, where $\mathbf{z} = col\{\mathbf{e}_1, \mathbf{x}_1, \mathbf{x}_1^*, \mathbf{r}\}$, $\boldsymbol{\theta}$ is a vector of tuned parameters. Select the deviation between the actual output subsystem's input $\mathbf{x}_2$ and virtual control of output subsystem $\mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta})$

$$\boldsymbol{\sigma} = \mathbf{x}_2 - \mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta}). \tag{4}$$

Introduce a tracking error dynamics between reference model (2) and subsystem $S_1$ of a plant (1)

$$\begin{aligned} \dot{\mathbf{e}}_1 &= \mathbf{f}_1(\mathbf{x}_1, \xi) - \mathbf{f}_1^*(\mathbf{x}_1^*) - \mathbf{g}_1^*(\mathbf{x}_1^*)\mathbf{r} + \\ &+ \mathbf{g}_1(\mathbf{x}_1, \xi)\left(\mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta}) - \boldsymbol{\sigma}\right). \end{aligned} \tag{5}$$

Find an "ideal" virtual control $\mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta}_*)$ which ensures the achievement of the control objective (3) assuming plant's (1) parameters are known. Note that $\boldsymbol{\theta}_* = \boldsymbol{\theta}_*(\xi)$ is vector of "ideal" virtual control parameters. Assume the following assumption is valid.

***Assumption D1.*** For any $\xi \in \Xi$ there are a) a vector $\boldsymbol{\theta}_* = \boldsymbol{\theta}_*(\xi) \in R^{m_\theta}$, b) continuous, bounded on arguments vector-function $\mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta}_*)$, and c) scalar continuous strictly growing function $\rho_Q(Q) > 0$ such that $\rho_Q(0) = 0$ and $w(\mathbf{e}_1, \boldsymbol{\theta}_*, 0) \le -\rho_Q(Q(\mathbf{e}_1))$, where

$$\begin{aligned} &w(\mathbf{e}_1, \boldsymbol{\theta}_*, \boldsymbol{\sigma}) = (\partial Q / \partial \mathbf{e}_1) \times \\ &\times \left(\mathbf{f}_1(\mathbf{x}_1, \xi) - \mathbf{f}_1^*(\mathbf{x}_1^*) - \mathbf{g}_1^*(\mathbf{x}_1^*)\mathbf{r} + \mathbf{g}_1(\mathbf{x}_1, \xi)\left(\mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta}_*) + \boldsymbol{\sigma}\right)\right) \end{aligned}$$

is the speed of change of function (3) along trajectories of system (5) assuming $\boldsymbol{\sigma} \equiv 0$. For brevity, drop arguments $\mathbf{x}_1, \mathbf{x}_1^*, \mathbf{r}$ in $w(\bullet)$.

***Note 1.*** Input-to-state stability of the reference model (2) implies the reference model is globally uniformly asymptotically stable. Therefore, the assumption D1 holds, for example, if unique solution $\mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta}_*)$ of the following equality exists

$$\mathbf{g}_1(\mathbf{x}_1, \xi)\mathbf{x}_2^{virt}(\mathbf{z},\boldsymbol{\theta}_*) = \mathbf{f}_1^*(\mathbf{e}_1) - \mathbf{f}_1(\mathbf{x}_1, \xi) + \mathbf{f}_1^*(\mathbf{x}_1^*) + \mathbf{g}_1^*(\mathbf{x}_1^*)\mathbf{r}.$$

***Stage 2***. Replace the unknown parameters $\boldsymbol{\theta}_*$ of the "ideal" virtual control by tunable ones $\boldsymbol{\theta}$. Design adaptation law for $\boldsymbol{\theta}$ ensuring achievement of the control objective (3) under parameter uncertainties $\xi \in \Xi$.

***Assumption C.*** The function $w(\mathbf{e}_1, \boldsymbol{\theta}, \boldsymbol{\sigma})$ is convex on $\boldsymbol{\theta}$, that is

$$w(\mathbf{e}_1, \boldsymbol{\theta}, \boldsymbol{\sigma}) - w(\mathbf{e}_1, \tilde{\boldsymbol{\theta}}, \boldsymbol{\sigma}) \le (\boldsymbol{\theta} - \tilde{\boldsymbol{\theta}})^{\mathrm{T}} \nabla_{\boldsymbol{\theta}} w(\mathbf{e}_1, \boldsymbol{\theta}, \boldsymbol{\sigma}),$$

where $\nabla_{\boldsymbol{\theta}} w(\mathbf{e}_1, \boldsymbol{\theta}, \boldsymbol{\sigma}) = (\partial w(\mathbf{e}_1, \boldsymbol{\theta}, \boldsymbol{\sigma}) / \partial \boldsymbol{\theta})^T$ is gradients of $w(\bullet)$ on $\boldsymbol{\theta}$.

Design the family of adaptation algorithms of virtual control parameters based on the speed-gradient method

$$d\left(\boldsymbol{\theta} + \boldsymbol{\psi}(\mathbf{e}_1, \boldsymbol{\theta})\right) / dt = -\boldsymbol{\Gamma} \nabla_{\boldsymbol{\theta}} w(\mathbf{e}_1, \boldsymbol{\theta}, \boldsymbol{\sigma}), \tag{6}$$

where $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^T > 0$ is a $(m_\theta \times m_\theta)$ positive defined gain matrix; $\boldsymbol{\psi}(\mathbf{e}_1, \boldsymbol{\theta}) \in R^{m_\theta}$ is a continuous vector-function satisfying the following assumption.

***Assumption*** $\boldsymbol{\Psi}$. $\boldsymbol{\psi}(\mathbf{e}_1, \boldsymbol{\theta})$ is a pseudo-gradient that is $\boldsymbol{\psi}(\mathbf{e}_1, \boldsymbol{\theta})^{\mathrm{T}} \nabla_{\boldsymbol{\theta}} w(\mathbf{e}_1, \boldsymbol{\theta}, \boldsymbol{\sigma}) \ge 0$. There exists a unique solution

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