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A Consensus Algorithm Based on Nearest Second-order Neighbors' Information ⋆

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Abstract: For the problem of communication in consensus algorithm based on second-order neighbors' information, a consensus algorithm using the nearest second-order neighbors' information, in which only partial second-order neighbors' information is used, is proposed and applied to continuous multiple motion bodies with double-integrator kinematics. We conducted the stable analysis to compare the proposed algorithm with the consensus algorithms based on the first-order neighbors' information, and based on the second-order neighbors' information on the convergence, respectively. The three consensus algorithms were used to multiple robot formation with leader-followers. The simulations have demonstrated the feasibility and validity of the proposed algorithm.

Keywords: Consensus algorithm, Second-order neighbors' information, Double-integration kinematics, Multiple robots, Leader-followers

1. INTRODUCTION

Recently, the coordination control of multi-robot systems has attracted the attention of the large number of scholars in engineering and academia due to several advantages such as redundancy, concurrency, robustness and its broad application in such areas including target search, cooperative object transport task, rescue missions and so on (Kim et al (2007) and Yang et al (2010)).

As one of important and fundamental search topics in coordination control of the multiple robot systems, consensus problem has been studied for a long time since the connection between the performance of a linear consensus protocol on a directed network and eigenvalues of the mirror graph of the information flow is established by Olfati-Saber and Murray (2004). Considering a modified leader-follower structure for the team, a semi-decentralized optimal control strategy was designed via minimization of individual cost functions over a finite horizon using local information to accomplish a cohesive motion with consensus on the agreed upon output by Semsar-Kazerooni and Khorasani (2007). In order to solve the finite-time consensus problem in a network of continuous time integrators with additive disturbances, a distributed algorithm employing the mixed use of continuous and discontinuous local interaction rules was proposed by Franceschelli et al (2013). For the consensus problem of multiple agents with continuous-time second-order dynamics where the sampling period of each agent is independent of the others' and the interaction topology among agents is time-varying, some new methods were provided to design controller gains and sampling periods to reach consensus by Gao et al (2011). A consensus algorithm was designed by a state feedback gain for the consensus problems for multi-input networked multi-agent systems with communication delay by Huan Pan et al (2012). The consensus condition for high-order multi-agent is derived from properties of M-matrix by adopting a vector Lyapunov function. Besides, the finite-time consensus problems of the linear and the nonlinear algorithm are also a hot topic ,such as in Sayyaadi er al (2011) . At first, first order consensus problem is often considered and relative to topology construction of communication topology graph. However, most multi-robot systems in reality are related to not only the position of the robot but also the velocity of the robot, the consensus algorithm of the multi-robot system with double-integrator dynamics or high-order needs to be developed, such as in Ren (2005).

In social activities, a common phenomenon is that people prefer to contact, corporate and make friends with friends of their friends comparing with strangers. It turns out that the second-order neighbors' information plays an important role in our life. Based on the concept, a consistency algorithm based on the second-order neighbors information is proposed by Jin and Murray (2006), and this algorithm is applied to the first-order system. Comparing with system only with first-order neighbours information, the convergence rate of this algorithm is faster. Furthermore, this method is applied to the second-order continuous, discrete-time and time-delay multi-agent systems by Pan (2012). Similarly with application in first-order system, it can also accelerate the convergence speed comparing with general linear consensus algorithm. However, the disadvan-

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tage of this algorithm is obvious. It increases the quantity of communication lines greatly. It assumed that if there are neighbors for a robot in multi-robot systems and each robot has the same number of neighbors, then the method will make the communication lines of each robot increase with the speed of geometric time square.

To address this question, combining with the advantages and disadvantages of the second-order neighbors' information, a consensus algorithm based on the nearest second-order neighbors' information is presented in this paper. For any robot in formation, the set of second-order neighbors that it communicates with consists of two parts. One is made up of the robot among the second-order neighbors which has the most important influence on the original robot. The other part is made up of the robots among the second-order neighbours that the original robot has the largest effect on. Furthermore, for the multi-robot system with double-integrator dynamics, a contain algorithm is also given and analyzed.

This paper is organized as follows. In Section II, some preliminaries about graph theory and matrix theory, especially the properties of graph Laplacians are briefly introduced. In Section III, a new consensus algorithm based on the nearest neighbors' information is proposed which can accelerate the convergence speed and decrease the burden of communication. In Section IV, to illustrate our theoretical results, some simulations including the generate algorithm, the algorithm based on second-order neighbours' information and the proposed algorithm are presented. Finally, we conclude the paper in Section V.

2. PRELIMINARIES

Graph theory is the basis knowledge of the research on consensus algorithm. A weighted directed or undirected graph G considered is defined as $G = (V, E, A)$, where $V = \{v_1, ...v_n\}$ is the set of nodes, $E \subseteq V \times V$ is a set of edges, $A = [a_{ij}]$ is a weighted adjacency matrix with nonnegative adjacency elements and is defined by $a_{ii} = 0, a_{ij} = w_{ij}$, if$(j, i) \in E$ and 0 otherwise. Moreover, n is the number of nodes, e_{ij} that belongs to set of edges E denotes the edges from node to which is also defined as $e_{ij} = v_i, v_j$. Graph can be divided into two categories, the directed graph and the un-directed graph. If the edges of graph are directed, the graph is directed. Then $(v_i, v_j) \in V \neq (v_j, v_i) \in V$. If the edges of graph are undirected, the graph is undirected. Then $(n_k, n_m) \in V = (n_m, n_k) \in V$.

The set of first-order neighbours of node v_i is expressed as $N_i = \{v_j \in V : (v_j, v_i) \in E\}$. The set of second-order neighbors of node v_i is defined as $N_i^2 = \{v_j \in N, v_k \in V : (v_k, v_j) \in E\}$ for all $k \neq i$. besides, the Laplacian matrix $L = [l_{ij}] \in R^{n\times n}$ is defined as $l_{ii} = \sum_{j=1}^{n} a_{ij}$, and $l_{ii} = -a_{ij}$ for $i \neq j$. For an undirected graph, the Laplacian matrix of it is symmetric, but for an directed graph , it is asymmetric.

3. CONSENSUS PROTOCOLS AND ANALYSIS

3.1 Consensus Protocols

We consider the multi-robot continuous systems, where each of them with double-integrator kinematics is described generally as follows

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \tag{1}$$

where $x_i(t)$ and $v_i(t)$ stand for the state and velocity of the robot respectively, u_i denotes the control input of this system. When the system (1) achieves consensus, which means that all robots' states and velocity converge to the same values. There exists

$$\lim_{t\to T} |x_j(t) - x_j(t)| = 0, \lim_{t\to T} |v_j(t) - v_j(t)| = 0 \quad (i, j \in W, i \neq j) \tag{2}$$

where W is the set of robots, and T is the time that robot achieves consensus. For system (1), a typical consensus algorithm with first-order neighbors' information (FNI) is described as

$$u_i(t) = \sum_{j\in N_i} a_{ij}[k_1((x_j(t) - x_i(t)) + k_2((v_j(t) - v_i(t))] \tag{3}$$

where a_{ii} denotes the weight of edge between robot i and robot j , which is also the (i, j) entry of the corresponding adjacency matrix. k_1 and k_2 denote the adjustable parameters. N_i presents the set of second-order neighbors of robot i . Because of the important role of the second-order neighbors information, the consensus algorithm based on second-order neighbours information (SNI) was proposed by Pan et al (2012) and applied to system with double-integrator. Its actual expression is expressed as

$$\begin{aligned} u_i(t) = &\sum_{j\in N_i} a_{ij}[r_1((x_j(t) - x_i(t)) + r_2((v_j(t) - v_i(t))] \\ &+ \sum_{k\in {N_i}^2} a_{ik}[r_1((x_k(t) - x_i(t)) + r_2((v_k(t) - v_i(t))] \end{aligned} \tag{4}$$

where a_{ik} denotes the weight of edge between robot i and robot k , which is also the (i, j) entry of the corresponding adjacency matrix . r_1 and r_2 denote the adjustable parameters. N_i and ${N_i}^2$ presents the set of first-order neighbors of robot i and the set of second-order neighbors of robot i respectively. Considering the problem of this control law and the advantages of second-order neighbours information, the consensus algorithm based on the nearest second-order neighbors' information (NSNI) that's proposed in this paper is expressed as

$$\begin{aligned} u_i(t) = &\sum_{j\in N_i} a_{ij}[k_1((x_j(t) - x_i(t)) + k_2((v_j(t) - v_i(t))] \\ &+ \sum_{m\in {N_i}^2} a_{ik}[k_1((x_m(t) - x_i(t)) + k_2((v_m(t) - v_i(t))] \end{aligned} \tag{5}$$

where ${N_i}^2$ presents the set of second-order neighbours that have most important effect on robot i or robot i have most effect on them.

3.2 Consensus Analysis

In this section, we discuss the performance of the proposed algorithm, especially the convergence rate. Firstly, a key