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IFAC-PapersOnLine 48-11 (2015) 563–567

Adaptive Stabilization of Minimum-Phase Plant under Lipschitz Uncertainty via Yakubovich's Method of Recurrent Objective Inequalities ★

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Abstract: This paper addresses a problem of adaptive stabilization of minimum-phase plant under Lipschitz uncertainty and bounded external disturbance. Solution of the problem is based on the method of recurrent objective inequalities proposed by V.A. Yakubovich in the 1960s. This method allows not only to stabilize unknown plant with the use of a simple projection type estimation algorithm, but to ensure online model validation as well.

Keywords: adaptive control, robust control, identification, uncertainty, bounded disturbance, model validation

1. INTRODUCTION

The term *adaptive control* is generally associated with meeting control objective via online estimation of some uncertainties in control system while the term *robust control* is associated with meeting control objective without online estimation of any specific uncertainties from a certain admissible prior set. Note that uncertainties of different types can be equivalent from robust control point of view. For one example, robust stability and robust performance conditions for systems with structured uncertainties are equivalent in the ℓ_1 control theory for linear time varying, nonlinear time invariant, and nonlinear time varying uncertainties (see Khammash and Pearson (1991) for detail). But from adaptive control point of view, the time invariant uncertainties differ appreciably from the time varying uncertainties because they do not exclude, in specific cases, the possibility of their online estimation for improvement of control performance.

The capabilities of feedback based on online estimation of nonparametric nonlinear time invariant uncertainty of Lipshitz type were first studied in Xie and Guo (2000) for the simplest dynamical model of the form

$$y_{t+1} = f(y_t) + u_t + w_{t+1}\,, \quad t = 0, 1, 2, \dots ,$$

where the real numbers y_t, u_t, w_t denote the output, control, and unknown bounded external disturbance at the time instant t, respectively. The unknown function $f : \mathbb{R} \to \mathbb{R}$ represents a Lipshitz uncertainty in the model:

$$|f(x_1) - f(x_2)| \leqslant L|x_1 - x_2| \quad \forall x_1, x_2 \in \mathbb{R}\,.$$

It was shown in Xie and Guo (2000) that the specific Lipshitz constant

$$L = 3/2 + \sqrt{2}$$

is *critical*. If $L \geq 3/2 + \sqrt{2}$ then for any (causal) feedback, that is for any control of the form

$$u_t = U_t(y_0, \dots, y_t, u_0, \dots, u_{t-1})\,, \quad t = 0, 1, 2, \dots ,$$

with any functions U_t and for any bounded sequence w there is a Lipschitz uncertainty f such that the output of the closed loop system is unbounded. Furthermore, a feedback that estimates nonparametric uncertainty f in closed loop and ensures asymptotically optimal tracking a given bounded reference signal for the model with the Lipschitz constant $L < 3/2 + \sqrt{2}$ was proposed in Xie and Guo (2000)).

More general model of the form

$$y_{t+1} = ay_t + f(y_t) + u_t + w_{t+1}\,, \quad t = 0, 1, 2, \dots$$

with the unknown coefficient a, the unknown Lipschitz constant L and unknown upper bound on the external disturbance w was considered in Sokolov (2003a,b) where the problem of suboptimal robust tracking a given bounded reference signal was solved under the assumptions $|a| \leqslant \overline{a}$ and $L \leqslant \overline{L}$ with the known upper bounds $\overline{a}$ and $\overline{L} < 3/2 + \sqrt{2}$.

All the more general model of the from

$$y_{t+1} = f(y_t) + g(\theta, \varphi_t, u_t) + w_{t+1}\,,$$

where g is a known function, θ is an unknown real vector from a known bounded parallelotope Θ, and $\varphi_t = (y_t, \dots, y_{t-n}, u_{t-1}, \dots, u_{t-m})$, was considered in Huang and Guo (2012). The problem of adaptive stabilization of this model was solved with the use of infinite memory feedback under following assumptions: an upper bound $\overline{L} < 3/2 + \sqrt{2}$ on the Lipschitz constant L is known; an upper bound on the external disturbance w is known; upper bounds on the partial derivatives of the function g are known; the model is minimum-phase (see Huang and Guo (2012) for detail). The unknown vector θ was estimated via the exhaustion over a sufficiently small grid in Θ (that is, over a finite subset of Θ which is δ-dense in

★ New estimation algorithm for nonlinear systems under Lipschitz uncertainty (Sec.4) was developed under support of Russian Scientific Foundation (grant 14-29-00142) in IPME RAS.

10.1016/j.ifacol.2015.09.246

Θ for certain small δ). Current estimates of the unknown vecrot θ were discarded if the output or the control violate certain inequalities associated with the current estimates and followed from the prior assumptions about the model. Such an estimation algorithm is mainly of theoretical interest.

In order to get a more practicable stabilizing feedback, we consider the standard auto regressive moving average model of the form

$$y_{t+1} = a_0 y_t + \cdots + a_n y_{t-n} + b_0 u_t + \cdots + b_m u_{t-m} + f(y_t) + w_{t+1}$$

where the unknown coefficient vector

$$\theta = (a_0, \ldots, a_n, b_0, \ldots, b_m)$$

is from a known convex closed bounded prior set Θ and the model without the Lipschitz uncertainty f is minimum-phase for all $\theta \in \Theta$. An upper bound on the external disturbance w is also assumed to be known.

While the method of online estimation of Lipschitz uncertainty in this paper is the same as in Xie and Guo (2000); Sokolov (2003a,b); Huang and Guo (2012), the estimation of the unknown coefficient vector θ is based on the main idea of the method of *recurrent objective inequalities* (ROI), which was initially proposed in Yakubovich (1966, 1968). Another titles of the ROI method in adaptive control literature are *recursive aim inequalities* and *recursive goal inequalities*. Applications of this method to adaptive control of systems under bounded disturbances and a vast literature on the method were presented in Fomin, Fradkov, and Yakubovich (1981) (in Russian; see also Bondarko and Yakubovich (1992) in English).

The application of the ROI method to adaptive control of more general class of systems under Lipschitz uncertainty and bounded external disturbance is possible thanks to the fact that the Lipschitz property of uncertainty is in the form of inequality. The benefits of this application are as follows. First, the ROI method allows to use a simple finitely convergent projection type algorithm for online estimation of the unknown coefficients θ and provides a faster convergence of estimates. Second, it introduces less conservatism in control than the estimation method in Huang and Guo (2012). Third, the ROI method provides a direct online verification of the estimated model by data.

As in Sokolov (2003a,b), we additionally ensure a finite memory of stabilizing feedback via partitioning the output space into intervals and storing only one of the outputs y_t in each interval. The less is the length of the intervals, the less is the conservatism introduced into the online estimation of the uncertainty f and the more is the computer memory required for storing data.

Notation

$\mathbb{N} = \{0, 1, 2, \ldots\}$ - the set of natural numbers;

$\mathbb{R}$ - the field of real numbers;

$x_p^q = (x_p, x_{p+1}, \ldots, x_q)$ – a piece of the real sequence $x = (x_0, x_1, \ldots)$;

$|x_p^q| = \max_{p \leqslant k \leqslant q} |x_k|$;

$|\varphi|$ – the euclidean norm of the real vector φ;

$\|G(\lambda)\| = \sum_{k=0}^{+\infty} |g_k|$ for stable transfer fuction $G(\lambda) = \sum_{k=0}^{+\infty} g_k \lambda^k$.

2. PROBLEM STATEMENT

Let a plant under control be modeled in the form

$$y_{t+1} = a(q^{-1})y_t + b(q^{-1})u_t + f(y_t) + w_{t+1}\,, \quad t \in \mathbb{N}\,, \quad (1)$$

where the real sequences y, u, and w denote the plant output, the control, and the external disturbance, respectively. The unknown function $f : \mathbb{R} \to \mathbb{R}$ represents the Lipschitz uncertainty in the model:

$$|f(x_1) - f(x_2)| \leqslant L|x_1 - x_2| \quad \forall x_1, x_2 \in \mathbb{R}\,. \quad (2)$$

The polynomials $a(q^{-1})$ and $b(q^{-1})$ in the backward shift operator q^{-1} represent the nominal model,

$$a(q^{-1})y_t = a_0 y_t + \cdots + a_n y_{t-n}\,,$$
$$b(q^{-1})u_t = b_0 u_t + \cdots + b_m u_{t-m}\,,$$

and the coefficients of the polynomials are unknown.

The prior information about the controlled plant is in the form of Assumptions **A1**, **A2**, and **A3**.

A1. The unknown coefficients of the polynomials $a(\lambda)$ and $b(\lambda)$ belong to a known convex compact set Θ,

$$\theta = (a_0, \ldots, a_n, b_0, \ldots, b_m) \in \Theta \subset \mathbb{R}^{n+m+2}\,, \quad (3)$$

and the polynomials $b(\lambda)$ are stable for all θ in Θ (that is, the roots of $b(\lambda)$ are outside the closed unit circle of the complex plane). Moreover, constants C_1, C_2, and C_3 are known such that

$$\left\|\frac{1}{b(\lambda)}\right\| \leqslant C_1,\ \|a(\lambda)\| \leqslant C_2,\ \|b(\lambda)\| \leqslant C_3 \quad \forall\, \theta \in \Theta\,. \quad (4)$$

A2. $L \leqslant \overline{L} < 3/2 + \sqrt{2}$ and the upper bound $\overline{L}$ is known.

A3. $\sup_{t \geqslant 0} |w_t| \leqslant \overline{W}$ and the upper bound $\overline{W}$ is known.

The problem under consideration is to design a finite memory feedback that stabilizes the plant under prior Assumptions **A1**, **A2**, and **A3**.

3. STABILIZING FINITE MEMORY FEEDBACK UNDER KNOWN COEFFICIENT VECTOR θ

For purposes of clarity, we first describe a finite memory feedback that stabilizes model (1) under the assumption of known coefficient vector θ. This feedback is similar to those in Sokolov (2003a,b) and is a simple modification of the basic control law in Xie and Guo (2000) according to Remark 2.5 there. Choose any positive number ε and and consider a partition of the real numbers

$$\mathbb{R} = \bigcup_{k \in \mathbb{Z}} [k\varepsilon, (k+1)\varepsilon)\,. \quad (5)$$

For a chosen feedback, denote by

$$(y_{k_0}, y_{k_1}, y_{k_2}, \ldots)$$

the subsequence of outputs that fall first into their intervals of the partition, that is,

$$y_{k_j} \in [k\epsilon, (k+1)\epsilon) \Rightarrow \forall t < k_j \quad y_t \notin [k\epsilon, (k+1)\epsilon)\,.$$

For any t define

$$i_t = \operatorname*{argmin}_{\{k_j | k_j < t\}} |y_t - y_{k_j}| \quad (6)$$