Measurement 48 (2014) 96–108

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

An optimization technique based on imperialist competition algorithm to measurement of error for solving initial and boundary value problems

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ARTICLE INFO

Article history:
Received 6 September 2013
Received in revised form 20 October 2013
Accepted 28 October 2013
Available online 9 November 2013

Keywords:
Imperialist competitive algorithm
Evolutionary algorithm
Initial and boundary value problems
Unconstrained optimization
Penalty method

ABSTRACT

Imperialist competitive algorithm (ICA) is proposed to solve initial and boundary value problems in this paper. A constrained problem is converted into an unconstrained problem through the use of a penalty method in other to define an appropriate fitness function that is optimized by means of the ICA method. The methodology adopted evaluates a large number of candidate solutions of the unconstrained problem with the ICA to minimize error measure, which quantifies how well a candidate solution satisfies the governing ordinary differential equations (ODEs) or partial differential equations (PDEs) and the boundary conditions. The method is proficient approach to solve linear and nonlinear ODEs, systems of ordinary differential equations (SODEs), linear and nonlinear PDEs. Numerical experiments demonstrate the accuracy and efficiency of the proposed method. Thus, this method is a promising tool for solving higher-dimensional problems.

1. Introduction

Initial-value and boundary-value (IVBV) problems are predominantly in most scientific disciplines and are principally common in engineering. These problems are characterized by governing ordinary differential equations (ODEs) or partial differential equations (PDEs) and their associated boundary and initial conditions. The result of solving a direct IVBV problem is either a functional relationship or a system of equations that can be applied to relate the values of the dependent variable to the values of the independent variables. From the time when IVBV was introduced by Laplace, Lagrange, and Euler [1], various methods of solving direct IVBV have been introduced. Other prevalent methods for solving ODE include the Runge–Kutta method, the predictor–corrector method, collocation, shooting [2], radial basis functions [3,4] and feed-forward neural networks [5–7]. Furthermore, newer methods which make use of genetic programming and methods that deduce the differential equation from experimental data was introduced in [8–12]. An array of many fields of science discipline such as physics, engineering chemistry and finance are undoubtedly uses partial differential equations (PDEs). Example of some significant PDEs are those of the theory of nonlinear conservation laws that are extensively applied in the modeling of water in unsaturated soil, the dynamics of soil water, the statistics of flow problems, mixing and turbulent diffusion, cosmology and seismology [13–15]. The extensive applications of PDE resulted in an all-important need to study this area in modern mathematical analysis and computational fluid dynamics. Nevertheless, to determine exact solutions for complex PDEs is hard problem. Multiple techniques for solving very simple or special-case PDEs exist in the literature. Some of these techniques include the finite element, finite difference, finite volume, and boundary element methods that are widely applied in the mechanical engineering fields to solve some specific PDEs [16–18]. The solution in these techniques is approximated by using a

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0263-2241/$ - see front matter
http://dx.doi.org/10.1016/j.measurement.2013.10.043

mesh or a grid of points in the search space. In recent times, a lots of research efforts have been dedicated to develop mesh-free algorithms in solving operator problems. Kansa [19,20] established that the classical collocation approach for solving PDEs solves a generalized interpolation problem. The findings by the author suggest the possibility of applying function approximation techniques in novel mesh-free numerical schemes for solving PDEs. It can be noted from what has been pointed out previously that solving high-order and nonlinear or complex PDEs is a hard problem. Exact solutions can be considered as unknown black boxes, which make it difficult to analyze the solution. Because the solution of a PDE is an unknown system, an approximate method capable of handling any unknown system can be used to find an appropriate solution for the PDE. In the work presented by Lagaris et al. [6], a technique to solve boundary value problems with irregular boundaries was proposed. The approach presented was centered around the synergy of two feed-forward ANNs of different types: a multilayer perceptron (MLP) is used as the basic approximation element, and a radial basis function (RBF) network is used to satisfy the boundary conditions (BCs). In [9], a combination of the genetic programming (GP) and least-squares collocation approaches have been applied to solve elliptic PDEs with the use of formulation comparable to the neural network-based collocation algorithm of Lagaris et al. [7]. The work is based on the concept of least-squares collocation principle which can be used to establish a fitness criterion that quantifies how well the expressions evolved by the GP method satisfy the governing equations and the boundary conditions. The fitness values for a set of collocation points are computed inside the domain and on its boundary. On the other hand, to decrease the size of the very large search space, the expressions evolved by the GP method are repaired; the GP model is hybridized with a radial basis function (RBF) network if the boundaries are geometrically complicated.

At present, evolutionary algorithms (EAs) have attracted much attention from researchers which are utilized in many applications [21–27]. Atashpaz-Gargari and Lucas [28] in 2007 proposed imperialist competitive algorithm (ICA) as one of the popular algorithms in the evolutionary computation domain. This algorithm is enthused by imperialism concept, in which some powerful countries try to take hold of others as there colonies. ICA is in recent times, has been successfully employed in many applications [29–34].

This paper proposed ICA technique to solve initial and boundary value problems. In the proposal, ICA is used as an optimization process that evolves a large number of candidate solutions. The ICA technique is used to solve ODEs and PDEs to generate solutions in the search space. In the first place, the original IVBV problem using discretization form is manipulated; secondly, with converting each IVBV problem as a constrained optimization problem to an unconstrained optimization problem by a penalty coefficient is used to create a fitness function. Then ICA minimizes error measure in the evolutionary process that quantifies how well a candidate solution satisfies the governing ODEs or PDEs and initial conditions. The distinction of the proposed algorithm from the standard numerical algorithms is the manner in which the behavior of a solution is controlled.

The remainder of this article includes the following sections: Section 2 presents the preliminary, we present in Sections 3 and 4, the proposed method and the experimental result, respectively. Finally, in Section 5, conclusion is drawn and ideas for future work are presented.

2. Preliminary

2.1. Imperialist competitive algorithm (ICA)

Recently ICA was introduced as a new evolutionary algorithm [28] and it has been applied for finding optimum solution in various applications. ICA is not enthused by a nature based evolution phenomenon like majority of the meta-heuristic algorithms neither it thus implements the social political process of imperialism and imperialistic competition. There are 7 steps to which ICA algorithm is characterized [23].

Step 1: Creating initial empire.

A member of population in ICA is known as "Country". In a dimensional optimization problem of $1 \times N_{var}$ – a country is depicted in an array as specified in the following equation:

$$\text{Country} = [P_1, P_2, \ldots, P_{N_{var}}]. \tag{1}$$

Two types of country exist, these are countries with lower values of cost function (best countries) referred to as "Imperialist" (N_{imp}), and others referred to as "Colony" (N_{col}). An imperialist takeover some of the "Colony" countries in respect to its power, in this scenario, a powerful imperialist possess more colonies and vice versa. Fig. 1 shows the takeover process which results in empire creation.

Step 2: Assimilation.

All colonies whose empire is the same are forced to move in the direction of their corresponding imperialist in the assimilation part. Fig. 2 depicts this movement accordingly. As it is clearly shown in Fig. 3, a colony approaches its related imperialist by x units where x has a uniform distribution as detailed in the following equation:

$$x \sim U(0, \beta \times d) \tag{2}$$

where β is a number not less than 1 and d is the distance between colony and imperialist. Furthermore, θ is the deviation needed to move colony on the way to imperialist, this is defined as follow:

$$\theta \sim U(\delta, -\delta) \tag{3}$$

where δ is an adjuster for the deviation from the original direction (Fig. 2). Hence, the process is repeated for all colonies in all empires.

Step 3: Revolution.

A revolution is the terminology of ICA to describe a swift alteration in a part of countries' socio-political characteristics. Revolution process has an essential character