TOWARD UNIFICATION OF CONSTRAINED MECHANICS AND VIRTUAL FIXTURES IN HAPTIC RENDERING

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Abstract: Virtual Fixtures (VF) are defined as haptic or visual aids in assisting users of virtual and tele-operation environments during various tasks. They have been mostly defined similar to physical fixtures which are used in performing daily tasks such as holding-on to the railing when climbing the stairs or using a ruler to draw straight lines. Such analogous definitions can be even extended to the case of learning certain manual tasks under supervision, e.g. when one holds the child's hand in order to teach the association between the mouse movements and positions of cursor on the screen. This paper presents how notions from the computational mechanics for solving constrained dynamical systems can be extended for stable implementation of a class of VF in a haptic rendered environment. It is shown that some of the solution methodologies from computational mechanics can have direct implication in haptic rendering of stiff-constrained environment. Examples of such implementations are also presented using a haptic device interacting with a class of VF.

Keywords: constrained mechanics, virtual fixtures (VF), computational mechanics, haptic rendering

1. INTRODUCTION

Interaction with the virtual mechanics-based environment originated from various areas. For example, in the area of manufacturing, assembly planning and interactive CAD-based models, it is sometimes required for the designer to place a component in the already designed environment (Galeano and Payandeh, 2005). In this case, the designer can manipulate an object through usage of a haptic device where the feeling of the interaction forces can be used to manipulate or guide the grasped part [2]. The haptic interaction can also be used in the dis-assembly planning where the repairing guidelines can be augmented with the position and orientation information for determining the trajectories for dis-assembling of a particular component within the existing components (Bayazit *et al.*, 2001). Another area is in training in a virtual simulators and tele-operation

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[2] http://www.inria.fr/rapportsactivite/RA2004/siames2004/vid160.html

environment. Through practice, the trainee needs to learn proper handling and motion of the tool for various tasks, e.g. surgical procedures(Parada and Payandeh, 2005) (Payandeh and Stanisic, 2002). Another area of application is in rehabilitation engineering where physically challenged individuals can interact with a virtual environment for enhancing their expected impedance model needed for various tasks (Pernalete *et al.*, 2005). Again, the user interacts with the scene through a haptic device where constraining forces can be generated in response to certain wanted or unwanted input trajectories.

Virtual Fixtures (VF) are defined as haptic and visual aids in assisting users of virtual or teleoperation environments during various tasks. For example, in interaction with such environment through a haptic device, the user can feel the reactive or guiding forces which can be generated due to the presence of VF.

A number of challenges must be considered when implementing VF. These can be: a) design of VF as a assembly of basic elements (Kuang *et al.*, 2004); b) definition of force fields associated with the VF; c) stable computational models of constraining and guiding forces; and d) development of stable hybrid control laws where the user can move between interconnected VFs. This paper presents some preliminary relationships which exist between a stable modeling and simulation of constrained multi-body systems and simulation and interaction with a class of VF in a haptic rendered environment. These initial findings can further be used to define a unified framework for developing and implementing VF.

This paper is organized as follows: section 2 presents stable formulation for constrained computational mechanics; section 3 presents the notion of virtual fixtures in a haptic rendered environment with some examples and section 4 presents discussions and concluding remarks.

2. AN OVERVIEW OF CONSTRAINED MECHANICS

Simulation of motion for multi-body systems has been a subject of research for a number of years. One of the main challenges has been the modeling and simulation of constrained multi-body systems where it is required to have a fast and stable computational model. There have been many progress in achieving fast, robust and real-time simulation of such systems, for example see: (Aghili and Piedeceuf, 2003) (Joli *et al.*, 1993) (Feng *et al.*, 2004) (Seguy *et al.*, 2003). This section presents some of the key modeling and simulation approaches which are related to implementation of a class of VF and their stable haptic interaction.

Let us consider a multi body system characterized by n-dimension generalized coordinate vector q, and $n \times n$-dimension square symmetric mass matrix $\mathbf{M}(q)$ and a n-generalized internal force vector (Coriolis, centrifugal or elastic forces) $f_{int}(q, \dot{q}$. If this system is subjected to the m holonomic constraints defined as:

$$\mathbf{\Phi}(\mathbf{q}) = 0, \quad (1)$$

Then the governing equations given by application of fundamental principle of mechanics (Newton-Euler, Lagrange equations, virtual power, ...) are:

$$\mathbf{M}\ddot{q} = f_q + f_{co}, \quad (2)$$

where we define:

$$f_q = f_{int}(q, \dot{q}) + f_{ext},$$

f_{ext} represents the generalized external force vector and f_{co} represents the generalized constraint force vector computed as:

$$f_{co} = -\mathbf{C}_q^T \lambda.$$

The term $\mathbf{C}_q^T$ is the transpose of the constraint Jacobian matrix defined as:

$$\mathbf{C}_q = \partial \mathbf{\Phi} / \partial q,$$

and λ is the m-dimension vector of Lagrangian multipliers that need to be determined. Equation (2) and (1) represent a mixed system of differential algebraic equations (DAE) in $\ddot{q}$ and λ. It is of interest to recall the geometric approach of such system given by (Blajer, 2002). At each time t, the constraining motion of a multi body system can be represented as the motion of a generalized particle P in a subspace H of a n-dimensional metric space E. The subspace is defined implicitly as a surface by the equation (1) and if all the holonomic constraints are independent, the dimension of H is $n - m$.

For computational purposes, instead of thinking about the constraints in terms of hard surfaces, it has been proposed to replace the *stiff* constraints by a strong force field in a neighborhood of H, directed toward the surface $\mathbf{\Phi}(q)$. The effect can be interpreted as the action of m elastic forces $\lambda_i = \kappa_i \Phi_i (i = 1, \cdots m)$ pointed in the constrained directions toward the respective constraints and proportional to the constraint violation (Blajer, 2002) Figure 1.

Figure 1 shows that any error and deviation from the constrained surface will result in the reaction force proportional to the spring constant. Assuming the same *stiffness coefficient* κ in all constrained directions, the elastic force are $\kappa\Phi$,