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An algorithm for improving the coefficient accuracy of wavelet packet analysis

Xintao Jiao [a,b,*], Kang Ding [a], Guolin He [a]

[a] South China University of Technology, Guangzhou 510640, PR China
[b] South China Normal University, Foshan 528225, PR China

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ABSTRACT

On the basis of analyzing the cause of coefficients distortion inhered in the existing wavelet packet transform algorithms, a new method is presented to rectify the distortion and improve the accuracy. Its principle is to implement compensation calculation by using the complementary characteristic of wavelet filters. The simulation result of a typical signal shows the method is effective in improving the accuracy of wavelet packet coefficients. The influences of the compensation calculation times and vanishing moments of the wavelet are analyzed. Theoretically, the maximum error could be reduced to 25% with one time compensation calculation. The results show that with two and four times compensation calculations, the error is reduced to 10% and 2% respectively. The experiment result of the vibration signal of a roller bearing with outer ring fault shows that the proposed method can effectively magnify the amplitude of the fault frequencies in practice.

1. Introduction

Over the past 20 years, wavelet transform (WT) has emerged as one of the most popular signal processing tools because of its many distinguished merits. As a new method for time–frequency signal analysis, WT has the local characteristic of time-domain as well as frequency-domain, and its time–frequency window is changeable. It is more suitable for processing the non-stationary signals than fast Fourier transform (FFT) and the short time Fourier transform (STFT) [1]. In recent years, WT-based methods have been widely used in many fields such as singularity detection, fault feature extraction and de-noising [2]. Unfortunately, its resolution in the high frequency region is quite poor due to the coarse decomposition of the high-frequency components in the signal [3]. Wavelet packet transform (WPT) is an expansion of classical WT. As a subtler multi-resolution analysis algorithm, it decomposes signals into multi-levels and provides the same frequency bandwidth in each level. As a result, WPT provides higher frequency resolution than WT [4]. As one of the most popular methods in signal processing, WPT has been applied in various fields successfully. Bin et al. [5] introduced a new approach based on wavelet packet decomposition (WPD) and empirical mode decomposition (EMD) to extract fault feature frequency for rotating machinery early diagnosis. The effectiveness of that method was proven by the result of the experimental tests. Yusuff et al. [6] combined WPD and support vector regression (SVR) to locate the faults on transmission. The result showed it can determine the fault rapidly. Wang and Li [7] used WPD to recognize the border monitoring sound and achieved convincing efficiency. Kotnik and Kacic [8] presented a new noise feature extraction algorithm using WPD and autoregressive modeling of a speech signal. Diego and Barros [9] proposed a method based on the use of WPT for time–frequency analysis of harmonic distortion

* Corresponding author. Address: School of Mechanical and Automotive Engineering, South China University of Technology, No. 381, Wushan Road, Tianhe District, Guangzhou 510640, PR China. Tel./fax: +86 20 87113220.

E-mail address: jiaoxintao2010@163.com (X. Jiao).

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in power systems. Wang et al. [10] used WPT to analyze the attenuation characteristics of acoustic emission signals. The results indicated that WPT was an effective tool in extracting the attenuation characteristics. Pan et al. [11] developed a new robust method by using WPT and support vector data description (SVDD). Morsi and Hawary [12] used WPT to process non-stationary signals measured from electric power system and achieved accurate results that were very close to the true values. Chen et al. [13] analyzed the impulsive energy of the vibration signal to detect the piston condition of water hydraulic motor by using WPT and Kolmogorov–Smirnov (KS) test. Vong and Wong [14] used WPT to analyze the engine ignition signal to extract the features of the ignition pattern.

Although the WPT has been widely used in signal processing, unfortunately due to the non-ideal frequency domain characteristic of wavelet filters that will be discussed in Section 2.2, there are some limitations inhered in the canonical WPT algorithm such as the aliasing, the disorder of the frequency bands and the distortion of the wavelet packet coefficients. These limitations may greatly affect the accuracy in the applications. Researchers presented several effective methods to eliminate the aliasing and disorder. Wang et al. [15] analyzed the frequency aliasing problem in wavelet decomposition and presented a modified wavelet decomposition algorithm to guarantee correct decomposition. In 2001, they proposed another improved wavelet packet analysis (WPA) algorithm combined with FFT to avoid the aliasing [16]. Yang and Park [17] presented an anti-aliasing algorithm for discrete wavelet transform (DWT) and the simulation results showed it was quite effective for avoiding aliasing. He introduced this anti-aliasing algorithm into WPT and eliminated the aliasing of WPT successfully [18]. Wickerhauser [19] proposed a method to correct the disorder of the bands by arranging them according to the specific order.

The coefficients of WPT are used in many fields [20–22], and play important parts in de-noising, signal compression and fault feature extraction. But as far as known, the distortion of the coefficients has not drawn enough attentions from the researchers. It may directly affect the accuracy in those applications, so it is necessary to develop a method for correcting the distortion.

The paper is organized as follows. In Section 2, the principle of the WPT is briefly introduced and the frequency domain characteristics of the wavelet filters are analyzed. After this, the limitations of the existing WPT algorithms are described in detail. In Section 3, a new algorithm is proposed for improving the coefficients accuracy of WPT. In Sections 4 and 5, the proposed method is applied to process the simulation signal and experiment signal to verify the effectiveness. At last, the conclusions are briefly drawn in Section 6.

2. Wavelet packet transform

2.1. The principle of WPT

The fast binary WPT decomposition algorithm of signal $f(t)$ is defined as [23]:

$$\begin{cases} d_0^0(t) = f(t) \\ d_{j+1}^{2i}(t) = \sum_k h_0(k-2t)d_j^i \quad i = 0, 1, \ldots, 2^j - 1 \\ d_{j+1}^{2i+1}(t) = \sum_k h_1(k-2t)d_j^i \end{cases} \tag{1}$$

where $h_0(n)$ and $h_1(n)$ are a pair of orthogonal mirror filters for decomposition: $h_0(n)$ is a low-pass filter and $h_1(n)$ is a high-pass filter. In time domain, they satisfy Eq. (2).

$$h_1(k) = (-1)^k h_0(1-k) \tag{2}$$

The reconstruction formula of WPT is:

$$d_j^i(t) = \sum_k g_0(t-2k)d_{j+1}^{2i} + \sum_k g_1(t-2k)d_{j+1}^{2i+1} \tag{3}$$

where $g_0(n)$ and $g_1(n)$ are a pair of orthogonal mirror filters for reconstruction: $g_0(n)$ is a low-pass filter and $g_1(n)$ is a high-pass filter. The procedures of wavelet packet decomposition and reconstruction are shown in Fig. 1a and b.

In order to achieve a perfect reconstruction, the wavelet filters must satisfy the conditions shown in Eqs. (4) and (5).

$$H_0(\Omega)G_0(\Omega) + H_1(\Omega)G_1(\Omega) = 1 \tag{4}$$

$$H_0(\Omega+\pi)G_0(\Omega) + H_1(\Omega+\pi)G_1(\Omega) = 0 \tag{5}$$

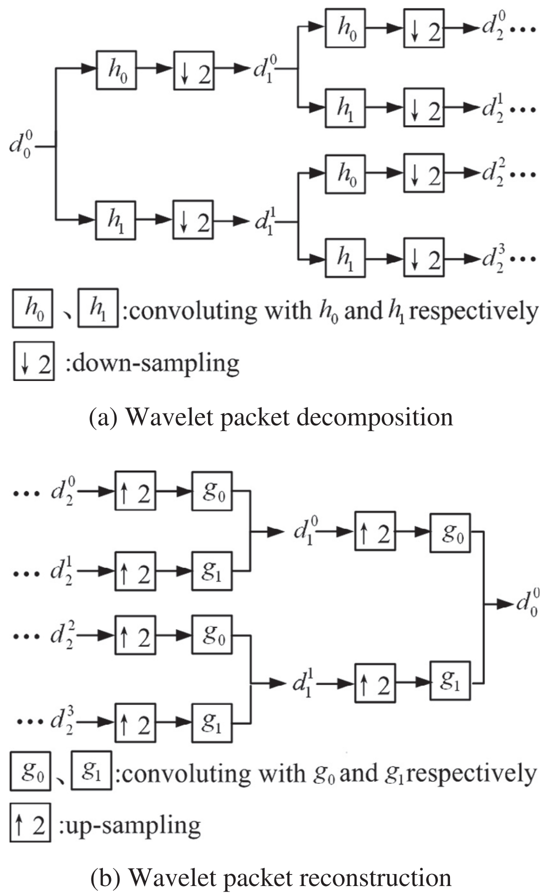

Fig. 1. Procedures of the canonical wavelet packet transform.