INVERTED PENDULUM ANGLE TRACKING CONTROL SUBJECT TO UNCERTAINTIES AND STOCHASTIC PERTURBATIONS

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Abstract: The tracking control problem of an inverted pendulum on a cart, operating under modelling uncertainties and stochastic perturbations is addressed. Suitable neural network designs and adaptive bounding algorithms are used to approximate all the unknown nonlinear uncertainties and stochastic disturbances. This scheme is integrated into the proposed nonlinear controller in order to achieve the angle tracking on a desired reference function. Stability analysis based on Lyapunov functions proves that all the error variables are bounded in probability; simultaneously, the mean square tracking error enters in finite-time in an arbitrarily selected small region around the origin wherein it remains thereafter. The controller performance is evaluated by simulation results. Furthermore, the design procedure and the effect of its parameters' selection are discussed. *Copyright © 2006 IFAC*

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1. INTRODUCTION

The inverted pendulum system is an inherent unstable system. Force must be properly applied to keep the system intact. To achieve this, proper control designs are required. However, the inverted pendulum system inherently has two equilibria, one of which is stable while the other is unstable. The stable equilibrium corresponds to a state in which the pendulum is pointing downwards. In the absence of any control force, the system will naturally return to this state. The stable equilibrium requires no control input to be applied and, thus, is uninteresting from a control perspective. The unstable equilibrium corresponds to a state in which the pendulum points strictly upwards and, thus, requires a control force to maintain this position. This is the basic control objective of the inverted pendulum problem. Furthermore, the tracking control objective of the inverted pendulum problem is to effectively follow a desired reference input.

Because of its special characteristics, the inverted pendulum system has been essentially used in evaluating and comparing various control theories. It is often used to demonstrate concepts in linear control. To this end, a linearized model is obtained about the unstable equilibrium. This model allows one to design a linear controller in order to balance the inverted pendulum around the upward equilibrium (Lin et al., 1996). However, due to the well-known drawbacks of the local linearization, nonlinear control techniques are proposed to be applied (Anderson, 1989, Angeli, 2001). Based on these designs, different systems consisting from an inverted pendulum attached on a cart equipped with a motor that drives it along a horizontal axis are proposed to illustrate the controller performance.

In this paper, such a system is considered which is modelled as an uncertain nonlinear system. A white noise is considered to act on this system due to unmodelled environment perturbations. Indeed, for output tracking control purposes of nonlinear unstable systems, the importance of taking into account the stochastic disturbances is evident. In the literature, theoretical methods are referred that confront separately either system uncertainties (Polycarpou, 1996, Spooner and Passino, 1996, Zhang, et al., 2000) or stochastic perturbations (Deng and Krstic, 1997, Liu and Zhang, 2004). A method that gives a solution to the combined problem is developed in Psillakis and Alexandridis (2006) for single-input single-output systems. Extending this method, an adaptive neural network (NN)-based motion controller is designed that effectively tracks the pole angle on a desirable reference function. To this end, the system variables are suitably transformed into error variables and appropriate Lyapunov functions are selected. In the sequel, proceeding with stability analysis, mainly based on these Lyapunov functions, the structure of the proposed tracking controller results. Particularly, integrating the control scheme to include suitable adaptive bounding algorithms, both the unknown nonlinear system uncertainties are approximated and boundedness in probability of all the error variables and the estimation errors is achieved. As it is proven in the paper the mean square tracking error enters in finite-time in a small region around the origin. Finally, the proposed controller is evaluated by simulation results wherein the excellent tracking response on a desired output reference is clearly verified.

2. SYSTEM MODELING AND PRELIMINARIES

2.1 Inverted Pendulum Model.

Consider the cart with the inverted pendulum (Fig.1). Assume that a horizontal stochastic perturbation acts on the center of gravity of the stick. Using Newton's laws, the dynamic nonlinear equations that describe the motion of the inverted pendulum on a cart can be obtained. Obviously, in state space form, a fourth order system is obtained with state variables: the pole angle and its derivative and the cart horizontal displacement and its derivative. However, since the precise horizontal position of the cart is of reduced importance, one can decouple the two first state equations from the last two equations by substituting the horizontal displacement and its derivative. It is proved, therefore, that eventually such a system can be precisely modelled by only two equations (Zak, 2003). However, in order to include the noise term, an equivalent form of these equations can be written as follows:

$$\begin{aligned} &dx_1 = x_2 dt\,, \quad y = x_1 \\ &dx_2 = f(x_1, x_2)\,dt + g(x_1)\,u dt + h(x_1)\,dw \end{aligned} \tag{1}$$

with

$$f(x_1,x_2) = \frac{g\sin(x_1) - \dfrac{mL}{2(m+M)}\sin(2x_1)\,x_2^2}{L\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{m+M}\right)}$$

$$g(x_1) = \frac{\dfrac{\cos(x_1)}{(m+M)}}{L\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{m+M}\right)} \tag{2}$$

and

$$h(x_1) = \frac{\dfrac{M\cos(x_1)}{m(m+M)}}{L\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{m+M}\right)} \tag{3}$$

where $x_1(t)$ and $x_2(t)$ denote the angular displacement θ and velocity $\dot{\theta}$ of the pole, respectively, $g = 9.8\, m/s^2$ is the acceleration due to gravity, M is the mass of the cart, m is the mass of the pole, L is the half-length of the pole and u is the applied control force. With w we denote a one-dimensional Wiener process defined on a probability space $(\Omega, \mathcal{F}, P)$ with incremental covariance $E(dw^2) = \delta dt$ such that $dw = n_w dt$ with n_w white noise process.

From the form of the function h one can conclude that there exists a nonnegative constant ψ_0 such that

$$|h(x_1)|^2 \le \psi_0 := \frac{\dfrac{M^2}{m^2(m+M)^2}}{L^2\left(\dfrac{4}{3} - \dfrac{m^2}{(m+M)^2}\right)} \quad \forall x_1 \in R$$

Assumption 1: The angular displacement is constrained in $|x_1| \le \pi/4$ so that

$$g(x_1) \ge \underline{g} := \frac{\sqrt{2}}{2L(m+M)}\left[\frac{4}{3} - \frac{m}{2(m+M)}\right]^{-1}$$

This can be made using some physical constraints in the design of the system (see Figure 1).

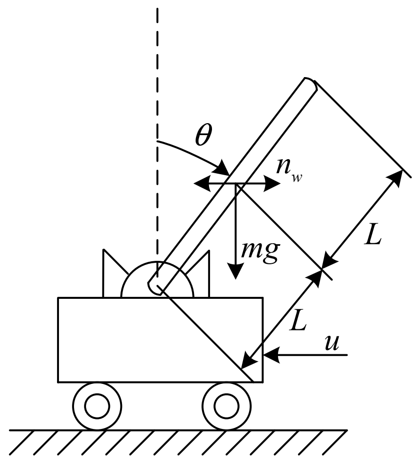

Fig. 1. The inverted pendulum on a cart.

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