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Equivalent electrical circuit framework for nonlinear and high quality factor piezoelectric structures[☆]

Tarcísio M.P. Silva[a], Marcel A. Clementino[a], Alper Erturk[b], Carlos De Marqui Jr[a,*]

[a] Department of Aeronautical Engineering, Sao Carlos School of Engineering, University of Sao Paulo, Sao Carlos, SP 13566-590, Brazil
[b] G. W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405, USA

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ABSTRACT

Equivalent electrical circuits are useful simulation tools to emulate and investigate the behavior of electromechanically coupled systems and structures as well as to develop energy harvesting or control circuits, among other configurations with two-way coupling. The existing efforts in this context have mostly considered linear structural (mechanical) domain, occasionally with nonlinear circuit for signal process and control. Typically, resonant circuits are employed to represent the mechanical domain of single- or multi-degree-of-freedom systems, while ideal transformers or current- and voltage-dependent sources are employed to model the electromechanical coupling. However, practical limitations of ideal transformers and dependent sources are challenges for experimental implementations of equivalent circuits. Furthermore, the internal resistance of equivalent resonant circuits limits the representation of high quality factor systems. This paper introduces equivalent electrical circuits for linear and nonlinear electromechanically coupled systems with high quality factor and various types of nonlinearities. The focus is placed on piezoelectric structures that exhibit stiffness and damping nonlinearities. An alternative to the existing models for the electromechanical coupling is also presented for convenient simulation of coupled system dynamics using standard electronic simulation programs. The equivalent circuit framework given here for high quality factor electromechanical systems is validated against both linear and nonlinear case studies, including published data for a nonlinear piezoelectric energy harvester. The proposed framework paves the way for the design of circuits emulating nonlinear structures, such as nonlinear vibration absorbers and sinks.

1. Introduction

Equivalent electrical circuit representations of electromechanical systems and structures have been used in numerous research domains from vibration control [1,2] to energy harvesting [3–6], while the present work is predominantly focused on piezoelectric energy harvesting and the literature review will be limited to that. The research field of vibration-based energy harvesting has been investigated by several research groups over the last decade [7–9]. The goal is to enable self-powered systems by converting the waste vibration energy available in their environments into usable electrical energy. Although different transduction mechanisms (piezoelectric [7,8,10–13], electromagnetic [14–18] and electrostatic [10,19,20]) have been suggested for converting vibrations to electricity, piezoelectric energy harvesting has received the most attention due to the high power density and ease of application of piezoelectric materials [8,21–23].

Linear resonant piezoelectric energy harvesters have been extensively studied and well understood [11,12], and more recently there has been growing interest in the modeling and leveraging mechanical nonlinearities [24] (which is separate from the leveraging of nonlinear electrical signal processing). The motivation in nonlinear energy harvesting is to exploit nonlinear phenomena to provide broadband energy harvesters, overcoming the main limitation of linear resonant energy harvesters (that are effective only at resonance excitation usually with a narrow bandwidth). The existing literature includes various nonlinear energy harvesters with intentionally designed nonlinearities [25–28] and several review papers [29–31] can be found for examples of nonlinear and other methods of broadband energy harvesting. In a recent effort, Leadenham and Erturk [32,33] presented the modeling and experimental validations of an M-shaped oscillator for broadband energy harvesting. The M-shaped configuration is an alternative to complex forms of symmetric Duffing oscillators and its flexible asymmetric

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[*] Corresponding author.

E-mail address: demarqui@sc.usp.br (C. De Marqui).

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nonlinear behavior yields broadband behavior under low excitation levels (as low as mill-*g* vibration levels). Furthermore, enhanced electrical power output was reported since the M-shaped structure made from spring steel is a high quality factor device (this strongly nonlinear configuration will be visited as a case study in the present work).

Most of the papers on linear and nonlinear energy harvesting have considered a simple load resistance in the electrical domain of the problem in order to estimate the electrical power output of the harvesters. However, charging a storage component requires a stable output voltage and efficient electrical circuits for linear and nonlinear energy harvesting devices. Another body of literature also includes standard AC-DC converters (as a one stage energy harvesting interface) [34], two-stage energy harvesting circuits that includes DC-DC conversion for impedance matching [35,36], and also synchronized switching circuits for piezoelectric energy harvesting [37,38]. Simulation of piezoelectric energy harvesters with more complex interface circuits requires implementing equivalent circuit models in circuit simulation software. Therefore, researchers have also presented equivalent circuit models for single- or multiple-degree-of-freedom (SDOF or MDOF) linear and nonlinear piezoelectric energy harvesting systems [3–6].

An early effort on the equivalent electrical representation of SDOF and MDOF linear electromechanically coupled systems was presented by Elvin and Elvin [3]. In another paper [4], the same authors described a coupled finite element – SPICE model of a linear electromechanically coupled system. Bayik et al. [6] presented an equivalent circuit for a piezo-patch energy harvester on a thin plate with AC-DC conversion. Equivalent circuit representations of the aforementioned papers [3–6] are limited to structurally (i.e. mechanically) linear systems with linear coupling since they are based on the modal decomposition of the DOFs of the electromechanically coupled system. To overcome this limitation, Elvin [5] presented two approaches to obtain the equivalent electrical representation of linear and nonlinear electromechanically coupled systems. First, system-level circuits approach is employed to model the harvester behavior. Then, dependent voltage equivalent circuits are employed to represent the nonlinear system.

Challenges remain for the practical implementation of equivalent circuits in the presence of nonlinearities. Most of the papers on equivalent electrical representation consider a resistive-inductive-capacitive (RLC) circuit as the equivalent of the mechanical domain of the problem while ideal transformers or voltage-dependent sources provide the forward and backward electromechanical coupling effects. The equivalent electrical representation of high quality factor mechanical and electromechanical systems would demand RLC circuits with extremely low internal resistance values (typically on the order of a few m Ω). In practice, however, such internal resistance is lower than the individual internal resistances of inductors and capacitors. Therefore, typical RLC circuits are unsuitable for the appropriate practical electrical representation of high quality factor devices. Furthermore, the representation of the electromechanical coupling effects by an ideal transformer cannot be obtained in practice, although they have been considered in standard electronic simulation programs.

In this paper, a novel nonlinear equivalent electrical circuit framework for SDOF linear and nonlinear oscillators with electromechanical coupling is presented with a focus on nonlinear piezoelectric structures. The circuit is based on operational-amplifier (op-amp) sub-circuits of low internal resistance to represent high quality factor system dynamics. The proposed framework also allows for the inclusion of different sources of nonlinearities, such as nonlinear damping, as well as symmetric and asymmetric nonlinear stiffness terms of any order. Moreover, a new equivalent electrical representation for the forward and backward coupling (electromechanical coupling of the system) is also presented. The validation of the proposed circuit is presented in two cases. First, the equivalent electrical representation of a high quality factor linear SDOF mechanical system (mass-spring-damper system) is considered. The experimental results obtained from breadboard implementation of the equivalent electrical circuit are compared with the numerical solution of the governing equation of the system. In the second case, the equivalent electrical representation for a SDOF nonlinear oscillator with electromechanical coupling is investigated. The experimental results obtained from breadboard implementation of the equivalent electrical circuit are validated against results from [33].

2. Components of the equivalent nonlinear circuit

The equivalent circuit is presented in this section as the electrical representation of a nonlinear SDOF oscillator with electromechanical coupling. Two different sources of nonlinearity are considered: nonlinear damping and nonlinear stiffness. Dissipative effects are assumed as a combination of linear viscous and quadratic damping (nonlinear dissipation) in the SDOF model. The other source of nonlinearity is nonlinear stiffness and the most general case of an *n*-order polynomial is considered in this section. The first-order coefficient is the linear stiffness for small displacements while the higher-order terms represent symmetric and asymmetric nonlinear stiffness terms. Therefore, the governing equations of the electromechanically coupled nonlinear oscillator are

$$m\ddot{z} + b_1\dot{z} + b_2(\dot{y} + \dot{z})|\dot{y} + \dot{z}| + F(z) - \theta V_p = -m^*\ddot{y} \tag{1a}$$

$$F(z) = k_1 z + \sum_{n=2}^{j} k_n z^n \tag{1b}$$

$$C_p V_p + Q_p + \theta z = 0 \tag{1c}$$

where m is the equivalent mass of the structure, m^* is the effective mass that causes the forcing term due to base excitation ($m = m^*$ if the spring mass is negligible when compared to the lumped mass attachments), b_1 is the linear viscous damping coefficient, b_2 is the quadratic (velocity-squared) damping coefficient (used to model dissipation due to fluid-structure interaction), $F(z)$ is the nonlinear elastic restoring force, where k_1 the linear stiffness coefficient and j is the order of the nonlinear stiffness, θ is the equivalent electromechanical coupling, V_p is the voltage across the piezoelectric material, Q_p is the electric charge output from the piezoelectric material, z is the relative displacement between the lumped mass and the moving base, and y is the base displacement as the source of mechanical excitation. An over-dot stands for differentiation with respect to time.

Equations (1) are the basis for the equivalent electrical representation of nonlinear electromechanically coupled systems presented in this work. Base excitation is the typical scenario in vibration energy harvesting, while other forms of mechanical excitation can be easily accommodated in other scenarios, such as vibration control, using the following framework. The solution of Equations (1) based on a system-level approach requires the following representation:

$$m\ddot{z} = -b_1\dot{z} - b_2(\dot{y} + \dot{z})|\dot{y} + \dot{z}| - F(z) + \theta V_p - m^*\ddot{y} \tag{2a}$$

$$V_p = -\frac{Q_p}{C_p} - \frac{\theta z}{C_p} \tag{2b}$$

The equivalent circuit representation of this system is displayed in Fig. 1 based on a force-voltage mechanical-to-electrical analogy for the special case of quintic (fifth order) nonlinearity. Therefore, the force terms of Eq. (2a) are implemented as voltages that are the input or output of each block in Fig. 1. Although similar system level equivalent circuits for nonlinear electromechanically coupled systems have already been presented in the literature [5], this section discusses and presents electrical circuit solutions that allow for the practical implementation and experimental validations of such systems. Moreover, the equivalent electrical circuit of the current paper also enables the electrical representation of high quality factor systems by keeping low internal resistances for each sub-circuit (each block) of Fig. 1. The