Mechatronics 53 (2018) 8–19

Contents lists available at ScienceDirect

Mechatronics

journal homepage: www.elsevier.com/locate/mechatronics

Fractional order sliding mode control via disturbance observer for a class of fractional order systems with mismatched disturbance,☆,☆☆

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ARTICLE INFO

MSC:
00-01
99-00

Keywords:
Fractional order calculus
Fractional order sliding mode control
Fractional order disturbance observer
Mismatched uncertain

ABSTRACT

This article proposes a novel fractional order sliding mode control for a class of fractional order and integer order systems with mismatched disturbances. First, a new fractional order disturbance observer is designed to estimate the fractional order differential of the mismatched disturbance directly. Second, the fractional order sliding surface and controller are proposed based on the designed disturbance observer. Our method can deal with mismatched disturbance and has better control performance with faster response speed, lower overshoot, and less chattering effect. The simulations on Quad–Rotor UAV and Maglev suspension systems demonstrate the effectiveness of the proposed method.

1. Introduction

Fractional calculus is a powerful method to describe data memory and heredity. It has the property of history dependence and long range correlation. It is the generalization of classical integer order calculus. It can describe some real systems more accurately than the traditional integer order method. Researchers have gradually found that fractional order calculus can characterize some non-classical phenomena in natural sciences and engineering applications. The theory of fractional calculus has been successfully applied in biology, physics, chemistry, automatic control, materials science, engineering, etc. [1,2]. As an important tool to improve the control performance, fractional calculus combines with many traditional control schemes, such as fractional order PID control [3], fractional order adaptive control [4], fractional order optimal control [5] and fractional order sliding mode control [6–8].

In real life, uncertainty and external disturbance often exist in the actual systems. External disturbances can be divided into two types: matching disturbances and mismatched disturbances. It is an important task to control the system and judge the stability of the system with mismatched disturbance. Sliding mode control (SMC) is an effective robust control method to deal with external disturbance. Several improved SMC methods have been proposed based on linear matrix inequality (LMI) [9], Riccati approach [10], adaptive technique [11], and neural network [12]. Integral sliding mode control (I-SMC) designs an integral sliding surface for a class of nonlinear fractional order systems [13,14], furthermore an improved stable sliding surface is given in [15]. Yet the closed-loop stability is not proven in these literatures. Lately the closed-loop stability is proven via the indirect Lyapunov method [16]. In I-SMC, a high-frequency switching gain is designed to force the states arrive the integral sliding surface. However, in the case of mismatched disturbances, I-SMC method can make states reach the desired equilibrium on the sliding surface. But at the same time, the I-SMC method may bring some adverse effects to the systems, such as large overshoot and long settling time. It is reported that the I-SMC method is applied to various systems [17,18].

When integer order SMC methods is used to deal with fractional order system, they always reject the disturbances in a robust way, but chattering is a serious problem that needs to be solved. [19–21] consider the chattering free control to avoid the serious chattering phenomenon in SMC. Kim [22] proposes a novel switching surface and a robust fractional control law. Subsequently, the sign function of the control input is transferred into the fractional derivative of the control signal in order to avoid the chattering. A new dynamic PID-SMC for a class of uncertain nonlinear systems is proposed and an adaptive parameter tuning method is used to estimate the unknown upper bounds of the disturbances [21]. This approach can eliminate the chattering phenomenon caused by the switching control action and

☆ This paper was recommended for publication by Associate Editor Prof. Hamid Reza Karimi.

☆☆ This work is supported by the National Natural Science Foundation of China (No. 61573050) and the open-project grant funded bythe State Key Laboratory of Synthetical Automation for Process Industry at the Northeastern University (No. PAL-N201702)

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https://doi.org/10.1016/j.mechatronics.2018.05.006
Received 13 December 2017; Received in revised form 28 March 2018; Accepted 12 May 2018
0957-4158/

realize high-precision performance.

Moreover, the disturbance observer (DOB) technique is also used to counteract the mismatched uncertainties and reduce chattering in the systems [22–25]. Yang et al. [26] proposes a disturbance observer based sliding mode control approach for systems with mismatched uncertainties. Li et al. [27] designs a sliding mode control based on nonlinear disturbance observer to counteract the mismatch disturbance and reduce the chattering. Zhang et al. [28] develops a disturbance observer -based integral sliding-mode control approach for continuous-time linear systems with mismatched disturbances. The disturbance observer is used to get the estimation of the disturbance, and the result can be incorporated in the controller to counteract the disturbance.

Considering the advantages of fractional order calculus, the fractional order is incorporated into the design of sliding mode control, which can improve the chattering problem and speed up the response of the closed-loop system. Wei proposed an adaptive backstepping output feedback control for a class of nonlinear fractional order systems [29]. In [30], the fractional order sliding mode control (FOSMC) for a single link flexible manipulator is realized. The control law of the proposed FOSMC scheme is designed using Lyapunov stability analysis. It has better control performance and is robust to external load disturbance and parameter variations. However, there is little research effort to combine FOSMC with fractional order disturbance observer, and then apply it to fractional order dynamic systems with mismatched disturbances. Pashaei and Badamchizadeh [31] tries to design a new FOSMC based on a nonlinear disturbance observer that exhibits better control performance, such as fast and robust stability, the disturbance and chartering rejection.

In this paper, we propose a new fractional order disturbance observer. It can estimate the fractional order derivative of the disturbance directly. The estimation of unmatched disturbance can be used to design sliding mode control law more conveniently. Subsequently, we designed a new fractional order siding mode control via a fractional order disturbance observer. The proposed FOSMC is generally applicable for both fractional order systems and integer order systems. The proposed method also shows good control performance and reduces the system chattering.

The main contributions of the paper are as follows. A new fractional order disturbance observer is proposed to estimate the mismatch disturbance and the estimation error is upper bounded. The main advantage is that it can get the fractional order of the mismatch disturbance directly. Based on the proposed DOB, a novel FOSMC method is proposed. It has better performances in weakening tracking error and chattering effect, compared with the traditional sliding mode control. The proposed FOSMC-DOB has less overshoot and faster convergence speed. Moreover, the proposed FOSMC-DOB is applicable not only for fractional order systems but also for integer order systems. In order to verify the excellent properties of the proposed FOSMC-DOB, a Quad-Rotor UAV system with integer order and a Maglev suspension system with fractional order are given. The proposed method shows good control performance in these systems. It can decrease the tracking error with a high rate of speed and weak chattering. Moreover, the FOSMC-DOB is not sensitive with controller parameters.

This paper is structured as follows. Section 2 gives the basic definitions of fractional calculus, the description of integer order and fractional order system. Section 3 contains two parts, one is the design of the fractional order DOB, and another is the design of FOSMC. Section 4 shows the experimental results for two different actual system. Finally, the conclusions are drawn in Section 5.

2. Basic knowledge and problem formulation

2.1. Basic definitions of fractional calculus

There are several definitions for fractional order derivatives [32], and three most commonly used definitions are the Grünwald–Letnikov's, Riemann–Liouville's, and Caputo's derivative definitions. In this paper, we use the Riemann–Liouville's definition.

Definition 1. The Riemann–Liouville fractional integral of α order of a continuous function $f(t)$ is defined as,

$$ {}_RD^{-\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}f(\tau)\mathrm{d}\tau,\quad t>t_0,\quad \alpha\in R^{+}, \tag{1}$$

where m is the largest positive integer number satisfying the following condition $m-1<\alpha<m$. Γ is the Gamma function, which is defined as following,

$$\Gamma(q)=\int_0^{\infty}x^{q-1}e^{-x}dx. \tag{2}$$

Definition 2. The Riemann–Liouville fractional derivative of α order of a continuous function $f(t)$ is defined as,

$$ {}_RD^{\alpha}f(t)=\frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\alpha)}\int_{t_0}^{t}\frac{f(\tau)}{(t-\tau)^{\alpha-m+1}}d\tau\right]. \tag{3}$$

It should be noted that the fractional integral of order $\alpha>0$ is represented by $D^{-\alpha}$.

Property 1. For $\alpha=n$, where n is an integer, the operation $D_t^{\alpha}f(t)$ is the same as the integer order calculus, i.e., $D_t^{n}f(t)=\frac{d^n}{dt^n}f(t)$, and also for α=0, we have $D_t^{0}f(t)=\frac{d^0}{dt^0}f(t)=f(t)$.

Property 2. The fractional order integration or differentiation calculus are linear operations, which is similar to the integer order calculus,

$$D_t^{\alpha}(\lambda f(t)+\mu g(t))=\lambda D_t^{\alpha}f(t)+\mu D_t^{\alpha}g(t). \tag{4}$$

Property 3 (*[33]*). For the arbitrary fractional order, $\alpha>0,\beta>0,\beta\in(m-1,m)$, the following equalities hold for the hybrid fractional derivative and integral operation,

$$D_t^{\alpha}(D_t^{\beta}f(t))=D_t^{\alpha+\beta}f(t), \tag{5}$$

$$D_t^{\alpha}(D_t^{-\beta}f(t))=D_t^{\alpha-\beta}f(t), \tag{6}$$

$$D_t^{-\alpha}(D_t^{\beta}f(t))=D_t^{-\alpha+\beta}f(t)-\sum_{j=1}^{m}\left[D_t^{\beta-j}f(t)\right]_{t=t_0}\frac{(t-t_0)^{\alpha-j}}{\Gamma(1+\alpha-j)}, \tag{7}$$

and $D_t^{\beta\text{-}j}f(t)$ are bounded at $t=t_0$.

Property 4 (*[33]*). The fractional derivative operator $D_t^{\alpha}f(t)$ commutes with $\frac{d^n}{dt^n}f(t)$, i.e., that

$$D^m(D^{\alpha}f(t))=D^{\alpha+m}f(t) \tag{8}$$

only if at the lower terminal $t=t_0$ of the fractional differentiation the function $f(t)$ satisfies the conditions

$$f^{(s)}(t_0)=0(s=0,1,\ldots,m-1). \tag{9}$$

Lemma 1 (*[19]*). *For an integrable function $f(t)$, if there is at least one $t_1\in(0,t)$ such that $f(t_1)\neq0$, then there is a positive constant N such that $D^{-\alpha}|f(t)|\geq N$.*

2.2. Problem formulation

Consider a general dynamical system with mismatched external disturbances,

$$\begin{cases}D^{\beta}x(t)=Ax(t)+Bu(t)+B_d d(t)\\ y(t)=Cx(t)\end{cases}, \tag{10}$$

where $\beta\in(0,1]$ is the order of the system, $x(t)\in R^n$ is the state variable, $u(t)\in R^m$ is the control signal, $y(t)\in R^p$ is the output and $d(t)\in R^1$ is the mismatched external disturbance. $A\in R^{n\times n}$ is the state matrix. $B\in R^{n\times m}$ is the control matrix. $B_d\in R^{n\times 1}$ is the disturbance matrix. $C\in R^{p\times n}$ is the output matrix. System (10) describes a fractional-order