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Modeling and simulating the thermoelastic deformation of mirrors using transient multilayer models☆

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ABSTRACT

We present a dynamic model with distributed parameters for the thermoelastic transfer behavior in multilayer structures, which are motivated by optically addressed deformable mirrors (OADMs). These are encountered in adaptive optics and utilized for correcting wavefront disturbances of high-power radiation. Our modeling approach is based on a continuum-mechanic multilayer model which distinguishes between an addressing heat load – the control input – and a boundary disturbance evoked by the high-power primary radiation. Thus, the model without control action can be used for passive mirrors as well. The relevant transient effects are investigated with physically motivated assumptions, the plate-like geometry, and parametric rheological analogue models. Furthermore, an efficient simulation scheme is established using Fourier methods in conjunction with model order reduction. The model's accuracy and the validity of all assumptions is demonstrated by means of an experimental setup. The parametric model is a first step towards feedback and feedforward control designs and disturbance compensation algorithms for OADMs.

1. Introduction

Deformable mirrors are used in adaptive optical in order to compensate for various optical disturbances. Initially, adaptive optics devices were introduced in the discipline of astronomy and aim at correcting wavefront distortions caused by astronomical scintillations or mechanical vibrations [1]. Thus, the image resolution of terrestrial telescopes can be significantly improved [2–4]. These comparably large mirrors ($D \geq 1$ m) are typically deformed by an array of voice coil actuators. More recently, the framework of adaptive optics is also used in ultraviolet lithography [5–7] or within high-power lasers [8–10]. In these applications the deformable mirrors have diameters up to a maximum of 50 mm. Hence, mechanical actuation principles cannot be realized with satisfactory spatial resolutions [6] and many degrees of freedoms at the same time, giving rise for optically addressed deformable mirrors (OADMs). The considered mirrors consist of three different layers: A filter glass with a high-reflective coating at the primary wavelength λ_δ, an acrylic glue intermediate layer, and a brass support. The latter one is actively cooled and acts as a heat sink. The coating at the top is transmissive at the second wavelength λ_u, which is addressed to the mirror from the top as sketched in Fig. 1. Since the secondary radiation is strongly absorbed, the addressing radiation $u(r, \varphi, t)$ is used as a volumetric heat load inside the OADM's filter glass substrate and triggers thermoelastic deformations $y(r, \varphi, t)$. However, a small fraction of the primary light is also absorbed in the coating of the mirror. At intensities of multiple kW/cm^2, the resulting heat flux is not neglectable and will evoke additional undesired deformations. In the context of high-power lasers, the heating of all involved optical elements is a well-known problem [11–13]. From this point of view, the incoming intensity $\delta(r, \varphi, t)$ has to be considered as a disturbance on the OADM.

A model with distributed parameters is introduced in this paper to represent the thermomechanical transfer behavior of the OADM which is subject to significant spatial dynamics. We consider a control loop which is typical for an adaptive optics setup (see Fig. 1). The control objective is to regulate the deformation of the OADM's surface $y(r, \varphi, t)$ (and thus the heat transfer) in the presence of the disturbance $\delta(r, \varphi, t)$ with the input $u(r, \varphi, t)$ being the distribution of the addressed intensity. To achieve virtually arbitrary input profiles, a spatial light modulator (SLM) is used. In this work, we focus on the parametric modeling and a real-time capable, efficient simulation algorithm of the relevant spatio-temporal transfer behavior.

If it is desired to apply the framework of automatic control to

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spatially distributed systems, the tradeoff between a precise model and the ability of real-time simulations is crucial [13–15]. In Section 2, we derive a constitutive material model in a three dimensional (3D) context within a continuum-mechanic framework [16–18]. The plate-like geometry of the filter glass is repeatedly exploited throughout the paper [19]. In Section 3, further physically motivated assumptions are imposed to the nonlinear partial differential equations (PDEs) in order to carry out the relevant transient effects. Thus, all constants which occur in the simplified model are parameterized by fundamental material constants like Young's modulus, the thickness of the individual layers or the specific heat capacity. The efficient simulation algorithm makes use of the system's structure and the chosen coordinate system. Fourier methods and a high resolution spatial discretization as presented in [20] are combined with model order reduction (MOR) techniques in order to balance the competing objectives 'spatial resolution' and 'real-time capability' [21–23]. This work extends the literature [7,24–27] in the following aspects:

- A parametric model based on fundamental continuum-mechanic balances is established and simplified by reasonable geometrical and physical assumptions.
- The introduced model considers two kinds of inputs: high primary light intensities $\delta(r, \varphi, t)$ which cause perturbing deformations as well as an addressing intensity $u(r, \varphi, t)$ generating desired deformations.

The benefit of a consistent and parametric modeling approach is that the impact of certain geometric or material parameters can be evaluated explicitly. Additionally, it possible to use the resulting model for investigating design issues, such as finding an optimum thickness of the individual layers. With an explicit representation of two input types u and δ, model-based compensation schemes can be addressed properly. Moreover, the model is not limited to active mirrors: With a vanishing control action $u \equiv 0$ the behavior of passive mirrors or similar optical devices under thermal load δ can be analyzed. On the other hand, issues like the processing of the reflected wavefront or details on the addressing unit's design are out of this contribution's scope. These issues have been rigorously discussed in the literature and can be combined with the presented model in a straightforward way [2,4]. Finally, we validate the introduced model by means of experiments reported in Section 4 and sum up the paper in Section 5 also sketching further research.

2. Material models for plate-like multilayers

In this section we derive the model equations for thermoelastic heat conductors consisting of multiple layers. The material model is introduced in Section 2.1 comprising a set of coupled PDEs which govern displacement $\boldsymbol{u}(\boldsymbol{x}, t) \in \mathbb{R}^3$ and absolute temperature $\theta(\boldsymbol{x}, t) \in \mathbb{R}^+$ in the solid material body. Afterwards, the general material law is applied to the plate-like top layer by distinguishing between in-plate and normal directions. Since the considered mechanical subsystem is linear, it is valid to discuss rheological concepts in the normal directions for the lower layers separately in order to determine the contact stresses in Section 2.2. Finally, the overall model is introduced by incorporating boundary conditions, controls, disturbances and measured outputs motivated by the setup in Fig. 1.

2.1. Constitutive in-plate material law

In the following, we derive a material model for thermoelastic heat conductors in a three-dimensional (3D) context. Note, that a summary of all formula symbols and adopted conventions is provided in Appendix A. After introducing the balance theorems, we discuss the constitutive equations which capture the standard Hookian law in the context of infinitesimal deformations as well as nonlinear effects arising

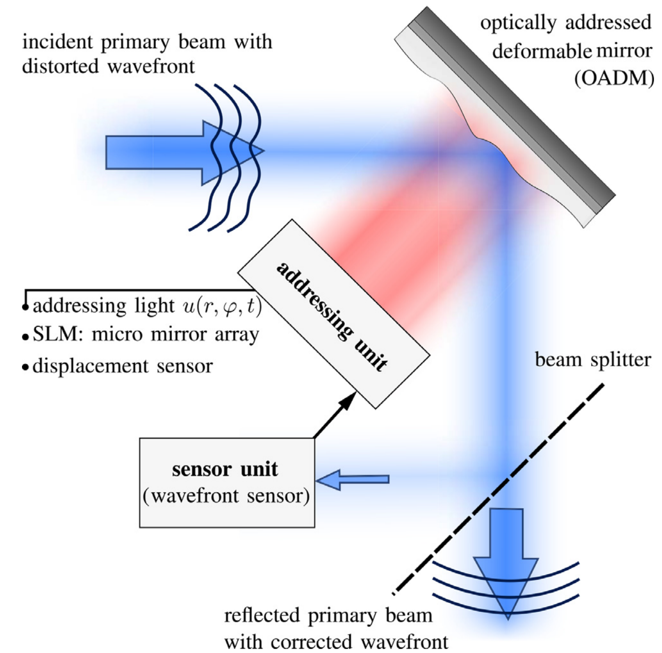

Fig. 1. Sketch of a typical control loop in adaptive optics. The incident primary beam has the wavelength λ_δ and the intensity $\delta(r, \varphi, t)$. The OADM is used to compensate for the distorted wavefront by addressing the extra intensity $u(r, \varphi, t)$ of wavelength λ_u into the OADM's filter glass. With a displacement sensor in the addressing unit, an internal feedback can be realized optionally. The wavefront of the reflected primary beam is measured in the sensor unit.

from finite thermal loads. We derive the temperature-deformation relations as a set of coupled PDEs by evaluating the balance theorems by means of the constitutive assumptions. Furthermore, the dissipation inequality is utilized to prove that the model equations are physically meaningful.

Balance theorems. The balance equations of a thermomechanical process comprise thermodynamic and mechanical states: Firstly, the Kelvin-temperature θ, enthalpy density η, heat flux $\boldsymbol{q}$ and Helmholtz enthalpy density ψ characterize the thermodynamics [28]. Secondly, the mechanical states density ρ, symmetric stress tensor $\boldsymbol{\sigma} = \boldsymbol{\sigma}^\intercal$, volume loads $\boldsymbol{g}$ and displacement $\boldsymbol{u}$ with respect to a reference position are considered. As a strain measure, the linearized Green–Lagrangian strain tensor

$$\boldsymbol{\varepsilon}(\boldsymbol{u}) = \frac{1}{2}(\nabla_3 \boldsymbol{u} + \nabla_3^\intercal \boldsymbol{u}) \tag{1}$$

is used [16]. The notation ∇_3 indicates the 3D gradient operator. Besides the balance of mass and moment of momentum,which are satisfied a priori in this context [28], the local balances for

$$\text{momentum:} \quad \rho \ddot{\boldsymbol{u}} = \nabla_3 \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g}, \tag{2}$$

$$\text{energy:} \quad \rho \dot{e} = \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} - \nabla_3 \cdot \boldsymbol{q} + \rho s, \tag{3}$$

$$\text{dissipation:} \quad 0 \leq \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} - \rho \eta \dot{\theta} - \rho \dot{\psi} - \theta^{-1} \boldsymbol{q} \cdot \nabla_3 \theta \tag{4}$$

are taken into account [17] with s and e being a volumetric heat source and the energy density. The above relations are under-determined but valid for any thermodynamical process independent of particular material properties. While the momentum and energy balances are used to derive the governing PDEs, the thermodynamic consistency can be checked by means of the dissipation inequality: It can be evaluated if the material model represents a physically meaningful behavior [28]. Consider the isotropic constitutive equations in dependence of strain $\boldsymbol{\varepsilon}$ and temperature θ

$$\boldsymbol{q}(\theta) = -\lambda \nabla_3 \theta \tag{5}$$