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Continuous sliding mode control of compliant robot arms: A singularly perturbed approach☆

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ABSTRACT

Compliant actuators are essential for ensuring safety in physical human-robot interaction. A compact-sized series elastic actuator (SEA), a type of compliant actuators, was developed to guarantee high force control fidelity and low output impedance in our previous work. This study is focused on the control design of a compliant robot arm driven by the developed SEA. The control problem is formulated into a singularly perturbed form which contains a slow rigid robot dynamics and a fast SEA dynamics. To achieve high-precision tracking without serious control chattering, a second-order sliding mode control (SMC) law is proposed such that the equilibrium point of the closed-loop rigid robot dynamics has semiglobal exponential stability. A derivative-type control law is employed such that the equilibrium point of the closed-loop SEA dynamics has global exponential stability. As the SMC law developed is continuous so that the rigid robot dynamics is continuously differentiable, the singular perturbation theory is applicable to establish semiglobal practical exponential stability of the entire system. This study provides the first application of continuous SMC to robots driven by compliant actuators. Experimental results have verified a high-accuracy and high-resolution tracking performance of the robot control system.

1. Introduction

Traditionally, robots are driven by stiff actuators to achieve stable, fast and accurate position control resulting in a critical safety problem when robots and humans share the same workspace [1]. This motivates the investigation of compliant actuators. A series elastic actuator (SEA) is a popular type of compliant actuators where passive elasticity is intentionally introduced in series between the motor and the load [2]. The SEA has several attractive features compared with stiff actuators, including low output impedance, back-driveability, shock tolerance, smooth force transmission, and energy efficiency [3]. However, compliant actuation also reduce system bandwidth, tracking accuracy, and stability margin [4], which brings difficulty for control design. After the seminal work [2], the SEA has made great advancement in recent years, where some typical results can be referred to [3–18].

A novel SEA was presented in our previous work [19]. A cable-driven version of this SEA, which mainly comprises a servomotor with two rotary encoders, a set of linear springs, a ball screw, and a potentiometer, is illustrated in Fig. 1. The motion from the motor is first transmitted to the ball screw via a coupler, which converts rotatory motion of the shaft to linear motion of a ball screw nut. The motion of the nut is transmitted to an output carriage through the linear springs, while the carriage drives a robot joint using a pair of cables. One encoder is applied to measure the angles of the motor and ball screw, another encoder is applied to measure the angle of the robot joint, and the linear potentiometer is applied to measure the displacement of the linear spring. This SEA possesses some attractive properties such as high force control fidelity, low output impedance, and compact size.

A variety of control strategies, including optimal control [19], disturbance observer-based control [15,20–22], sliding mode control (SMC) [23], and adaptive control [4,24], have been applied to SEAs or SEA-driven robots for physical human-robot interaction. However, all these approaches only deal with low-level actuator force/impedance control based on the SEA model, where the rigid robot model is regarded as unmodeled dynamics. It is shown in [25] that SEA-driven robots should be described by a flexible-joint robot (FJR) model with both the SEA model and the rigid robot model. To our knowledge, there exists only one result that considers the rigid robot model during the control design of SEA-driven robots so far [26]. In [26], a singular perturbation-based adaptive controller was developed for a lower-limb

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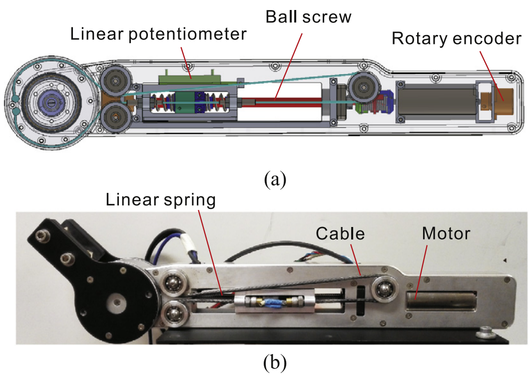

Fig. 1. An illustration of the cable-driven SEA mechanism. (a) A CAD model showing the design principle. (b) A prototype of the SEA.

training exoskeleton driven by SEAs, where the rigid robot model and the SEA model (i.e. the singular perturbation part) are considered to be slow and fast time-scale dynamics, respectively. An adaptive control law and a derivative-type control law are designed for the rigid robot model and the SEA model, respectively, and the *Tikhonov's theorem* [27] is invoked to establish the equilibrium point stability of the closed-loop singular perturbed system in [26]. The most salient feature of singular perturbation-based control is that the two controllers designed are parallel so that the control design can be greatly simplified and the complexity of the entire control law can be considerably reduced [28]. However, from the analysis of [29], the approach of [26] has a critical stability problem as the *Tikhonov's theorem* invoked is only valid for finite time so that tracking error convergence cannot be guaranteed in theory. The aim of this study is to develop a proper control strategy for a compliant robot arm driven by the SEA illustrated in Fig. 1. The control design is based on the following considerations: (1) Among all existing FJR control strategies, singular perturbation-based control has the lowest implementation cost as its controller complexity has the same order as the rigid robot counterpart [30]; (2) SMC provides high-precision tracking while rejecting both structured and unstructured uncertainties [31]; (3) only continuous SMC is applicable as the closed-loop rigid robot dynamics is required to be continuously differentiable for applying the singular perturbation theory [27]. By increasing the order of the plant dynamics, a second-order SMC (SOSMC) technique provides a feasible way to design continuous SMC [32]. Specifically, a robust integral of the sign of the error (RISE) feedback controller based on the SOSMC technique has been extensively studied to achieve asymptotic tracking of a class of nonlinear Euler-Lagrange systems [33–37]. However, only a few SOSMC designs consider singularly perturbed systems [38,39], and SOSMC has not been applied to FJRs or SEA-driven compliant robots. In [38], a class of nonlinear singularly perturbed systems is decomposed into slow and fast subsystems, and suboptimal SOSMC laws are designed for both the subsystems. In [39], a dynamic compensator-based SOSMC law is designed for a class of mechanical systems, and the control performance is analyzed by regarding the closed-loop system as a singularly perturbed system. Note that only simulation results are provided in [38,39].

In this paper, a singular perturbation-based continuous SMC strategy is proposed for the SEA-driven compliant robot arm in the laboratory. The control problem is formulated into a singularly perturbed form which contains the slow rigid robot model and the fast SEA model. To overcome the stability problem in [26], the *extended Tikhonov's theorem* [27] that is valid for infinite time is applied to analyze the singular perturbed system, where this theorem has an additional strong condition that the equilibrium point of the rigid robot model is exponentially stable. Based on the SOSMC technique, a continuous SMC law is developed to guarantee semiglobal exponential stability of the rigid robot model. Then, a derivative-type control law is applied to guarantee global exponential stability of the equilibrium point of the SEA model. Finally, the extended *Tikhonov's theorem* is applied to establish semiglobal practical exponential stability of the equilibrium point of the closed-loop singular perturbed system. *The contributions of this study include: (1) It gives the first application of continuous SMC to robots with compliant actuators; (2) experiments are provided to verify effectiveness of the applied controller.*

In the rest of this paper, the control problem is formulated in Section 2, the control synthesis and analysis is presented in Section 3, experiments are shown in Section 4, and conclusions are summarized in Section 5. Throughout this paper, $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the spaces of real numbers, positive real numbers, real n-vectors and real $m \times n$-matrices, respectively, $\min\{\cdot\}$ and $\max\{\cdot\}$ denote the minimum and maximum operators, respectively, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimal and maximal eigenvalues of A, respectively, L_∞ is the space of bounded signals, $\|\boldsymbol{x}\|$ is the Euclidean norm of $\boldsymbol{x}$, and col$(\boldsymbol{x}, \boldsymbol{y}) := [\boldsymbol{x}^T, \boldsymbol{y}^T]^T$, where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{y} \in \mathbb{R}^m$ and n and m are positive integers. In the subsequent sections, the arguments of a function may be omitted while the context is sufficiently explicit.

2. Problem formulation

A compliant robot arm driven by SEAs can be described by the following FJR model [25]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + D\dot{\boldsymbol{q}} + G(\boldsymbol{q}) + \boldsymbol{\tau}_d = K(\boldsymbol{\theta} - \boldsymbol{q}), \tag{1}$$

$$J\ddot{\boldsymbol{\theta}} + K(\boldsymbol{\theta} - \boldsymbol{q}) = \boldsymbol{u}, \tag{2}$$

where $\boldsymbol{q}(t) \in \mathbb{R}^n$ is a vector of joint angular position, $\boldsymbol{\theta}(t) \in \mathbb{R}^n$ is a vector of motor angular position, $M(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ and $C(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \mathbb{R}^{n\times n}$ are inertia and centripetal-Coriolis matrices in rigid links, respectively, $D\dot{\boldsymbol{q}} \in \mathbb{R}^n$ and $G(\boldsymbol{q}) \in \mathbb{R}^n$ are viscous friction and gravitational torques in rigid links, respectively, $J \in \mathbb{R}^{n\times n}$ and $K \in \mathbb{R}^{n\times n}$ are inertia and stiffness matrices of SEAs, respectively, $\boldsymbol{\tau}_d(t) \in \mathbb{R}^n$ is a perturbation, $\boldsymbol{u}(t) \in \mathbb{R}^n$ is a control torque, and n is the number of links. Let $\boldsymbol{q}_d(t) \in \mathbb{R}^n$ be a desired output. In this study, the following properties are available for the subsequent control design.

Property 1 [36]: *$M(\boldsymbol{q})$ is symmetric positive-definite and satisfies $m_0\|\xi\|^2 \le \xi^T M(\boldsymbol{q})\xi \le \overline{m}\|\xi\|^2$, $\forall \xi \in \mathbb{R}^n$, where $m_0, \overline{m} \in \mathbb{R}^+$ are some constants.*

Property 2 [36]: *$\dot{M}(\boldsymbol{q}) - 2C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is skew-symmetric such that $\xi^T(\dot{M}(\boldsymbol{q}) - 2C(\boldsymbol{q}, \dot{\boldsymbol{q}}))\xi = 0$, $\forall\, \xi \in \mathbb{R}^n$, which implies the internal forces do not work.*

Property 3 [36]: *If $\boldsymbol{q}, \dot{\boldsymbol{q}} \in L_\infty$, then $M(\boldsymbol{q})$, $C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and $G(\boldsymbol{q})$ are bounded and the first and second partial derivatives of $M(\boldsymbol{q})$, $C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and $G(\boldsymbol{q})$ with respect to their own arguments exist and are bounded.*

Property 4 [36]: *$\boldsymbol{\tau}_d$, $\dot{\boldsymbol{\tau}}_d$ and $\ddot{\boldsymbol{\tau}}_d$ exist and are bounded.*

Property 5 [36]: *$\boldsymbol{q}_d$, $\dot{\boldsymbol{q}}_d$, $\ddot{\boldsymbol{q}}_d$, $\dddot{\boldsymbol{q}}_d$ and $\ddddot{\boldsymbol{q}}_d$ exist and are bounded.*

Property 6 [25]: *D, K and J are positive-definite, diagonal and constant.*

Define a position tracking error $\boldsymbol{e}(t) := \boldsymbol{q}_d(t) - \boldsymbol{q}(t)$, a filtered tracking error $\boldsymbol{e}_f(t) := \dot{\boldsymbol{e}}(t) + K_f\boldsymbol{e}(t)$, an auxiliary variable $\boldsymbol{q}_r(t) := \dot{\boldsymbol{q}}_d(t) + K_r\boldsymbol{e}(t)$, and a virtual torque $\boldsymbol{\tau}(t) := K(\boldsymbol{\theta}(t) - \boldsymbol{q}(t))$, where $K_f, K_r \in \mathbb{R}^{n\times n}$ are prespecified positive-definite matrices. Our objective is to design a proper control strategy for the system comprised of (1) and (2) under Properties 1–6 such that $\boldsymbol{q}$ follows $\boldsymbol{q}_d$ well.

3. Singularly perturbed control design

3.1. Singular perturbation formulation

The spring stiffness K is expressed as $K = K_0/\varepsilon^2$, where $\varepsilon \in \mathbb{R}^+$ is a small parameter, and $K_0 \in \mathbb{R}^{n\times n}$ is a positive-definite diagonal matrix