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A gain scheduled robust linear quadratic regulator for vehicle direct yaw moment Control[☆,#]

Zhengyuan Wang[1,a], Umberto Montanaro[1,a,*], Saber Fallah[a], Aldo Sorniotti[a], Basilio Lenzo[a,b]

[a] Centre for Automotive Engineering, University of Surrey, Guildford GU2 7XH, UK
[b] Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield S1 1WB, UK

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ABSTRACT

Direct yaw moment controllers improve vehicle stability and handling in severe manoeuvres. In direct yaw moment control implementations based on Linear Quadratic Regulators (LQRs), the control system performance is limited by the unmodelled dynamics and parameter uncertainties. To guarantee robustness with respect to uncertainties, this paper proposes a gain scheduled Robust Linear Quadratic Regulator (RLQR), in which an extra control term is added to the feedback contribution of a conventional LQR to limit the closed-loop tracking error in a neighbourhood of the origin of its state-space, despite the uncertainties and disturbances acting on the plant. In addition, the intrinsic parameter-varying nature of the vehicle dynamics model with respect to the longitudinal vehicle velocity can compromise the closed-loop performance of fixed-gain controllers in varying driving conditions. Therefore, in this study the control gains optimally vary with velocity to adapt the closed-loop system to the variations of this parameter. The effectiveness of the proposed RLQR in improving the robustness of a classical LQR against model uncertainties and parameter variations is proven analytically, numerically and experimentally. The simulation and vehicle test results are consistent with the formal analysis proving that the RLQR reduces the ultimate bound of the error dynamics.

1. Introduction

Modern vehicle dynamics control systems are critical to the enhancement of lateral vehicle stability and the reduction of fatal accidents. In particular, vehicle control systems based on direct yaw moment control (DYC) enhance stability during cornering through the difference of traction and braking forces among the left and right wheels. DYC can be actuated through the friction brakes, torque-vectoring differentials, or individually controlled electric motors. The DYC actuation through the friction brakes is desirable only in emergency conditions, as it causes vehicle velocity reduction, and consequently degrades drivability and comfort. On the other hand, torque-vectoring differentials are characterized by significant mechanical complexity and actuation delays. The DYC implementation through individually controlled motors is more effective, because of the precise torque controllability and fast dynamics of electric machines [1–4].

Typically, DYC systems adopt a hierarchical control structure, consisting of three separate layers, namely the high-level controller, the mid-level controller, and the low-level controllers. The high-level controller is responsible for the reference generation at the vehicle level, and usually outputs the reference yaw moment for the mid-level controller, which distributes the torque demands among the available actuators (e.g., the electric motors and friction brakes), to generate the reference yaw moment and overall vehicle torque demand. The low-level controllers are responsible for the actuation of each individual component, based on the respective reference signals from the mid-level controller [5].

Different control techniques, such as model predictive control [6,7], robust control [8–10] and sliding mode control [11,12], have been proposed in the literature for the high-level controller. Linear Quadratic Regulators (LQRs) are among the most common control structures for DYC. To enhance the tracking performance for a wide range of longitudinal velocities, the solution of the Jacobi–Riccati equation of the LQR optimisation was exploited in Refs. [13–15] to formulate variable feedback and feedforward gains as functions of vehicle speed. However, the closed-loop stability of the resulting control systems was not

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* Corresponding author.
E-mail addresses: zhengyuan.wang@surrey.ac.uk (Z. Wang), u.montanaro@surrey.ac.uk (U. Montanaro), s.fallah@surrey.ac.uk (S. Fallah), a.sorniotti@surrey.ac.uk (A. Sorniotti), basilio.lenzo@shu.ac.uk (B. Lenzo).
[1] The first two authors, Zhengyuan Wang and Umberto Montanaro, equally contributed to this research.

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systematically investigated for time varying velocities. Furthermore, LQRs suffer from limited gain margin against parameter variations and external disturbances [16,17]. The robustness of the LQR implementations depends on the selection of the weights in the cost function to be minimised, which also affect the closed-loop response [16]. Usually, such weights are the result of time-consuming trial-and-error procedures to find a satisfactory trade-off between robustness and performance [18]. Alternatively, to enhance system robustness without increasing the design complexity, LQRs have been augmented with Variable Structure Control (VSC) actions. For example, a robust sliding mode yaw rate controller was proposed in [15] to address the tracking problem under uncertain conditions. Ref. [19] presents a sliding mode controller with time-varying sliding surfaces to solve the optimal control problem for both linear and nonlinear systems. Liu et al. [20] developed a LQR/VSC method based on the Planes Cluster Approaching Mode (PCAM) to guarantee global asymptotic stability in presence of parameter perturbations and unmodelled dynamics. However, despite their theoretical effectiveness in suppressing bounded disturbances, the discontinuous control terms, typically embedded in sliding mode controllers, induce chattering on the control action. In automotive applications, chattering may result either in stress and wear of mechanical and electrical parts, or in undesired vibrations during normal operation [21]. In addition, if the discontinuous control action is smoothed to mitigate chattering, often it is not possible to prove the asymptotic convergence to zero of the tracking error, but only its boundedness [21].

Considering these challenges, this paper proposes a novel approach to improve LQR robustness in DYC applications, against model uncertainties, real-time system parameter variations, and disturbances. This allows confining the tracking error in a pre-assigned neighbourhood of the origin, despite the time-varying nature of the longitudinal velocity, without adding discontinuous actions. More specifically, the proposed control action consists of three terms: (i) a feedback contribution whose gain is derived by solving the algebraic Riccati equation; (ii) a feedforward contribution based on the reference trajectory; and (iii) a feedback robust control contribution to improve closed-loop robustness with respect to unmodelled dynamics and parameter uncertainties. All control gains are functions of the longitudinal velocity for optimal tuning for a wide range of speeds. Therefore, the controller belongs to the class of gain scheduled Robust Linear Quadratic Regulators (RLQRs). The proposed RLQR also allows the decoupled design of the LQR and robust contributions, thus avoiding time-consuming tuning procedures for the selection of the LQR weights, which can be chosen without considering model approximations and disturbances. Then, based on the Riccati solution, the robust term is designed to suppress uncertainties. The closed-loop tracking error dynamics are analytically proven to be globally uniformly ultimately bounded. An upper limit for the ultimate bound (i.e., the maximum residual error when time tends to infinity [22]) is formulated, by considering the plant as a parameter-varying system [23]. Hence, unwanted dynamics, which can be induced by gain scheduling strategies [24], cannot emerge. The ultimate bound is inversely proportional to the gain of the robust contribution, which confirms the benefit of the proposed feedback structure. For its numerical validation, the novel RLQR is embedded in the IPG CarMaker simulation model of a prototype electric Range Rover Evoque with individually controlled motors on the front and rear axles. A quantitative comparison shows that the novel RLQR outperforms the gain scheduled LQR in [13], in terms of residual tracking error, peak yaw rate error and absolute value of the control action. Experimental results on the same electric vehicle confirm the applicability and effectiveness of the control strategy.

The paper is organized as follows. Section 2 describes the vehicle model for control system design and reference generation. Section 3 focuses on the control problem definition and control law formulation, while Section 4 deals with the analysis of the closed-loop tracking error dynamics through a Lyapunov approach. A vehicle simulation analysis for different manoeuvres is carried out in Section 5, while Section 6 discusses the implementation and performance of the controller on the case study electric vehicle demonstrator. Conclusions are summarised in Section 7, together with possible future developments.

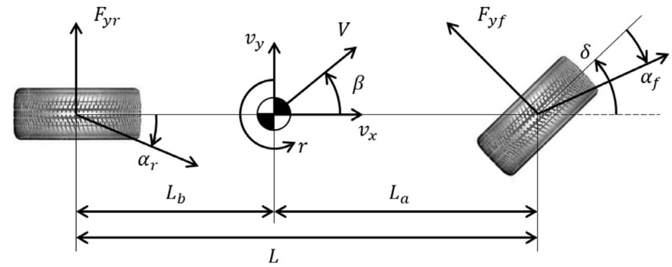

Fig. 1. The two-degree-of-freedom bicycle model.

2. Vehicle system modelling and reference behaviour design

This section formulates the model for control system design and an appropriate set of reference signals, based on the vehicle handling and stability characteristics. To this aim, the bicycle vehicle model, shown in Fig. 1, is used. In the figure δ is the steering angle, v_x and v_y are the longitudinal and lateral components of vehicle velocity, F_{yf} and F_{yr} are the front and rear lateral tyre forces, r is the vehicle yaw rate, β is the vehicle sideslip angle, and L_a and L_b are the front and rear semi-wheelbases. Despite its simplicity, the model reproduces the main handling and stability characteristics of a vehicle during cornering. Hence, it is often used in the literature in the control design stage.

The equations of motion are:

$$m(v_x\dot{\beta} + v_x r) = F_{yf} + F_{yr}, \tag{1}$$

$$I_z\dot{r} = L_a F_{yf} - L_b F_{yr} + u, \tag{2}$$

where I_z is the yaw mass moment of inertia, m is the vehicle mass, and u is the direct yaw moment, i.e., the control input. Since the actuator bandwidth is usually much larger than that of the closed-loop system [1,13,15], its dynamics are neglected in the control system design phase. Furthermore, in accordance with [13,14], a linear approximation of the lateral forces is used, thus:

$$F_{yf} = C_{\alpha f}\alpha_f, \tag{3}$$

$$F_{yr} = C_{\alpha r}\alpha_r, \tag{4}$$

where $C_{\alpha f}$ and $C_{\alpha r}$ are the cornering stiffness of the front and rear axles, and α_f and α_r are the front and rear slip angles, given by:

$$\alpha_f = \delta - \beta - tan^{-1}\frac{L_a r}{v_x} \approx \delta - \beta - \frac{L_a r}{v_x}, \tag{5}$$

$$\alpha_r = -\beta + tan^{-1}\frac{L_b r}{v_x} \approx -\beta + \frac{L_b r}{v_x}. \tag{6}$$

By combining (1)–(6), the state-space formulation of the vehicle model can be expressed as:

$$\dot{x} = Ax + Bu + E\delta, \tag{7}$$

where u is the control yaw moment, $x = [\beta \quad r]^T$ is the system state vector, while the system matrices are: