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Vehicle dynamics modelling and validation for online applications and controller synthesis

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ABSTRACT

For online automotive applications, such as vehicle dynamics control systems or state estimation, the choice of suitable mathematical models is important. In addition to model validity requirements, they have to fulfil requirements like limited computation time or numerical robustness. In this work two online suitable vehicle dynamics models with different modelling depths are discussed. First, a lumped mass model with a simplified suspension modelling technique and a complex tyre model is presented. A flatness based feed-forward and a simple feedback controller are used in order to control the vehicle speed. Additionally, a sensitivity analysis of the suspension modelling parts of the vehicle dynamics is presented. Besides the lumped mass model, a simplified model with the focus on lateral dynamics applications with a simple tyre model is introduced. The numerical properties of both models are analysed with respect to their time parameters at different speeds. An Audi A7 test vehicle equipped with precise measuring units is used in order to validate the models in the time and frequency domain. The lumped mass model shows superior validation results in lateral steady state and dynamical manoeuvres up to 2.5 Hz. The reduced model is also precise for up to 1 Hz and for steady state manoeuvres, which is sufficient for many lateral dynamics applications.

1. Introduction

Since the invention of passenger road vehicles, a lot of scientific and industrial work has been undertaken in order to understand and improve vehicle dynamics. Many aspects have been investigated regarding passenger ride comfort or driving safety.

In order to achieve these goals, not only the passive components of passenger vehicles like dampers or suspension joints have been optimized, but several active vehicle control systems have been developed. Examples in the vertical direction are variable damper or active suspension control systems [1–3]. In the horizontal direction an increasing number of systems like the anti lock braking [4], the electronic stability program [5], active steering systems [6,7] or torque vectoring systems [8–11] have been developed. Additionally, nowadays many advanced driver assistance systems are implemented in numerous vehicles. Examples are the active cruise control [12], the lane keeping assistant or the parking assistant [13].

These applications lead to the necessity of vehicle models, either specialized for the longitudinal, lateral or vertical dynamics, or a combination of these types due to integrated vehicle controllers [14]. These specialized vehicle models are usually used for controller synthesis like model predictive control schemes [7], dynamic feed-forward control [15] or state estimation [16,17]. So they have to describe the relevant effects of the system, but also have to be implemented on a target hardware with restricted memory, running robust and in real-time. This is, why the choice of the modelling depth and the selection of the degrees of freedom (DOF) of the model is important in online applications. The numerical behaviour of the model needs to be robust and a stable implementation on the real-time target must be guaranteed. In addition, for state estimation applications, for example using an extended kalman filter, the models should be continuously differentiable.

Simulation techniques have become more and more important in the optimization process of vehicle components in order to shorten the development process. So many vehicle models are used for validation simulations of control systems [18] or suspension systems. These models are complex and highly precise in order to match the real vehicle dynamics best possible. Therefore, many investigations on vehicle modelling have been done. One important group of models are the multi-body-models [19]. In [20] and [21] an overview of analysis methods and vehicle modelling techniques for handling simulations [22] is given. However, the computation time of the validation models is usually long,

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their code is not open for the typical user and they are usually too complex for online applications [23]. Consequently, the needs for simpler and also valid models arises.

For simulations up to the adhesion limit, one crucial component of a vehicle model is the tyre model for the calculation of the horizontal tyre forces with its strongly non-linear behaviour. Excessive studies were performed and several tyre models have been developed for vehicle handling simulations [24]. One widely used model is the Magic Formula Tyre model [25,26]. This model can describe the tyre behaviour precisely if numerous parameters are identified using tyre test rig measurement data [27].

Whereas complex and high precise vehicle and tyre models are studied extensively in the literature, investigations of simplified vehicle dynamics models of different modelling depths for online applications or controller synthesis as well as their validation with experimental data are rare. Therefore, the aim of this paper is to provide vehicle dynamics models of different modelling depth, which fulfil the requirements for online applications and are simple enough for controller synthesis purposes. Several modelling techniques for the tyre dynamics avoiding singularities are discussed. The longitudinal components of the vehicle are replaced by a speed controller with the ability of directly influencing the vehicle speed and avoiding the modelling effort for the engine, the transmission and the clutch. In addition, the significance of several suspension kinematics and elastokinematics is investigated. Subsequently, a simplified model for lateral dynamics applications with a minimum number of DOFs and states is presented. In order to reduce the number of parameters and computation time, a simple combined tyre model is used and adapted which is also analytically invertible and useful for control system synthesis.

This paper is organized as follows. In Section 2, a 14 DOF lumped mass model with a speed controller, tyre dynamics, suspension kinematics and elastokinematics is presented. A sensitivity analysis of the latter is investigated, the numerical properties are discussed and the calculation time on a real-time hardware is measured. A reduced 3 DOF model with the focus on lateral dynamics and a simplified combined tyre model is presented in Section 3. The model time parameters are discussed. The models are validated in Section 4 by means of steady state and dynamical test manoeuvres with an Audi A7 upper class vehicle in the time and frequency domain. Finally, a conclusion is given in Section 5.

2. A lumped mass full vehicle dynamics model for online applications

The aim of this section is to present an online suitable vehicle dynamics model which describes the vehicle dynamics in the horizontal as well as in the vertical direction. A compromise between model validity, modelling depth and realization effort is selected in order to fulfil the requirements of online applications like dynamic feed-forward control applications, simple real-time driving dynamics simulations or hardware in the loop tests.

2.1. Selection of DOFs

The model consists of five masses numbered with 0 to 4. Mass 0 belongs to the body of the vehicle and 1 to 4 to the unsprung masses of the wheels. Six degrees of freedom (DOF) completely describe the motion of the vehicle body. For description of the vertical eigenfrequency of the wheels and the vehicle longitudinal dynamics, each wheel has 2 DOFs. The first one belongs to the rotation, the second one to the vertical motion of the unsprung wheel mass. This configuration represents a lumped mass model [21]. Other DOFs of the tyres are neglected or treated as an input variable, e.g. the toe angle of each wheel.

Table 1
Variable conventions.

Symbol	Explanation	Symbol	Explanation
p	Position	kin	Kinematic
v	Velocity	sdak	Sum of s, d, arb, kin
a	Acceleration	bsh	Bushing
F	Force	κ	Longitudinal slip
M	Torque	α	Side-slip angle
ω	Angular velocity	m	Mass
$\boldsymbol{T}$	Transform. matrix	J	Inertia
r	Radius	wc	Wheel carrier
s	Spring	st	Suspension travel
d	Damper	sc	Speed controller
arb	Anti-roll bar	cp	Contact patch

2.2. Coordinate systems and variable conventions

In this model, several coordinate systems are used. The inertial system is I, the vehicle body system is V, the tyre contact patch fixed systems are T_i for each tyre $i = 1\ldots4$ as described in Fig. 1(a). The orientation of the vehicle body V is expressed with cardan angles in a cardanic system K.

The orientation of the coordinate systems is shown in Fig. 1(a), whereas the z-axis of the tyre systems are pointing towards the reader and are parallel with the z-axis of the inertial system.

Table 1 shows the variable conventions used in this paper based on [23]. Vectors or matrices are represented by bold symbols with their coordinate system at the upper left, e.g ${}^V\boldsymbol{p}_{0,1}$ means the position vector from the centre of gravity of body 1 in relation to the centre of gravity of body 0 represented in the coordinate system of the vehicle body V.

2.3. Transformation matrices

In order to clearly rotate the vectors from one system into another, transformation matrices are used. The transformation matrices from the vehicle system into the tyre systems ${}^{T_i}\boldsymbol{T}_V$ as well as from the vehicle system into the inertial system ${}^I\boldsymbol{T}_V$ can be found in [23]. The transformation matrix ${}^K\boldsymbol{T}_V$ from [23] for the angular velocities in the vehicle system V into the cardanic system K is not orthonormal and has a singularity in case of a vehicle pitch angle $\theta_0 = \pm\frac{\pi}{2}$. However, this case can be omitted in normal driving situations.

2.4. Differential equations

The translatory and rotatory equations of motion for the vehicle body, described in the vehicle system [23], are shown in equation

$$m_0\left({}^V\dot{\boldsymbol{v}}_0 + {}^V\boldsymbol{\omega}_0 \times {}^V\boldsymbol{v}_0\right) = \sum_{i=1}^{4} {}^V\boldsymbol{F}_{T_i} + {}^V\boldsymbol{F}_W + {}^V\boldsymbol{F}_g \tag{1}$$

and

$$\boldsymbol{J}_0 {}^V\dot{\boldsymbol{\omega}}_0 + {}^V\boldsymbol{\omega}_0 \times \left(\boldsymbol{J}_0 {}^V\boldsymbol{\omega}_0\right) = \sum_{i=1}^{4} {}^V\boldsymbol{M}_{T_i} + {}^V\boldsymbol{M}_W. \tag{2}$$

The main forces ${}^V\boldsymbol{F}_{T_i}$ caused by the tyres and described in (9), the wind force ${}^V\boldsymbol{F}_W$ and the gravity force ${}^V\boldsymbol{F}_g$ represented in the vehicle system are inputs for the translatory differential equation (1). ${}^V\dot{\boldsymbol{v}}_0$ is the time derivative of the vehicle body velocity vector. The most important torques introduced into the vehicle chassis ${}^V\boldsymbol{M}_{T_i}$ are caused by the tyre forces. They are described in detail in (15). The wind force is the reason for the wind torque ${}^V\boldsymbol{M}_W$ in (2). Both equations are coupled with the vehicle body angular velocity vector ${}^V\boldsymbol{\omega}_0$.