Mechatronics 000 (2016) 1–9

Contents lists available at ScienceDirect

Mechatronics

journal homepage: www.elsevier.com/locate/mechatronics

A magnetic flux model based method for detecting multi-DOF motion of a permanent magnet spherical motor

Kun Bai[a], Kok-Meng Lee[a,b,*], Jinjin Lu[a]

[a] *State Key Lab of Digital Manufacturing Equipment and Technology, School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China*
[b] *Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA*

ARTICLE INFO

Article history:
Received 20 October 2015
Revised 22 April 2016
Accepted 13 June 2016
Available online xxx

Keywords:
Back-emf
Spherical motor
Permanent magnet motor
Motion sensing

ABSTRACT

This paper presents a new method, referred to here as back-EMF method, for sensing the multi-DOF motion of a permanent magnet spherical motor (PMSM). With an explicit model characterizing the magnetic flux in the PMSM, a relationship between the electromotive force (EMF) induced in the winding of electro-magnets and the motion of the rotor permanent magnets is derived; and closed-form solutions that solves for multi-DOF Euler angles and angular velocities of the rotor are presented. This method allows for simultaneous estimation of both quantities in real-time by only measuring the voltages across the electro-magnets. Requiring no installation of additional sensors or fixtures on the rotor, the back-EMF method retains the structural simplicity of the PMSM. This method has been experimentally investigated on a prototype PMSM; the estimated Euler angles and angular velocities compare favorably with measurements from a commercialized gyroscope. As an immediate application, the motion states acquired using the back-EMF method are used for parameter estimation of the PMSM.

1. Introduction

Multi degree-of-freedom (DOF) electromagnetic actuators have been widely and increasingly employed in emerging applications such as robotics [1–4], air-space [5–6], and haptic devices [7]. A flurry of research has been directed towards developing novel multi-DOF actuators for smooth dexterous manipulation. Among the developments is the brushless permanent magnet spherical motor (PMSM) [8–12] capable of offering three-DOF motion in a mechanically simple ball-like joint with a wire-free rotor. The structural simplicity of PMSMs has motivated many researchers to seek precise manipulation of PMSMs where multi-DOF motion sensing plays an important role in order for closed-loop control.

The orientation sensing of PMSMs has been achieved through a variety of techniques. In [13], a customized mechanism was designed to mechanically decouple the PMSM motion into three independent directions for measuring with three single-axis encoders. The motion-constraining mechanism introduces additional inertia and friction not only limiting the PMSM bandwidth but also causing physical wear and tear. Inclinometers, accelerometers, and other inertia and gyroscopic sensors offer an alternative means to measure the orientation/position. However, these sensors which must be directly attached on the moving body introduce additional inertia and dynamical imbalance to the system; and additional modules/bridges are also required for power and signal transmissions. There are also non-contact solutions for orientation sensing which include optical [14], vision [15] and magnetic-field-based methods [16–17]. However, the structural and computational complexities involved in these sensing systems have often restricted the control performances of PMSMs.

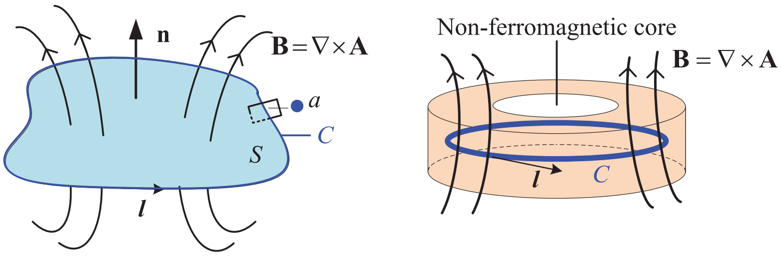
Many modern control methods that can be applied on PMSMs [18–19] usually require all system states (both orientation and angular velocity). Direct derivatives of Euler angles introduce noises in the measurements and thus affect control performance. State observers applied in many motion systems can estimate system states, however, require a precise model of the system. It is desired that the angular velocity can be detected directly with physical models. Back-electromagnetic force (back-EMF) method that characterizes the relationship between induced winding voltage and magnetic flux change has been utilized in many single-axis motors as a sensing technique as well as sensor-less means for control [20–22]. As the relative motion between permanent magnets (PMs) and electromagnets (EMs) in PMSMs leads to magnetic flux changes in the EM windings, there have been techniques developed for motion sensing of PMSMs in a back-EMF manner, where the magnetic flux linkage in stator EMs were derived using Finite

---

[*] Corresponding author: Fax: +1(404)894-9342.
E-mail addresses: kbai@hust.edu.cn (K. Bai), kokmeng.lee@me.gatech.edu (K.-M. Lee).

http://dx.doi.org/10.1016/j.mechatronics.2016.06.001
0957-4158/© 2016 Elsevier Ltd. All rights reserved.

Please cite this article as: K. Bai et al., A magnetic flux model based method for detecting multi-DOF motion of a permanent magnet spherical motor, Mechatronics (2016), http://dx.doi.org/10.1016/j.mechatronics.2016.06.001



(a) Closed-loop conductor (b) MFL in an EM

Fig. 1. Illustration of magnetic flux computation.

Element Analysis [23] and summation of average flux [24]. However, PMSMs present a unique challenge to develop an accurate and time-efficient model for the EMFs induced by the three-DOF motion.

This paper offers an explicit model for the magnetic flux linkages in a PMSM by utilizing geometric symmetries of both PMs and EMs which are common in PMSM designs [3, 5, 7–9]. Closed-form solutions to the orientation and angular velocity for real-time sensing of a PMSM are presented. The remainder of this paper offers the followings:

(1) The relationship between the EMF induced in an EM winding and rotational motion of a PM for an EM-PM pole pair is formulated. The model is then extended to multiple pole-pairs of EMs and PMs, upon which the induced EMFs are presented in explicit functions of the Euler angles and angular velocities for a PMSM; and the solutions to its inverse model are derived in an incremental manner.

(2) The back-EMF method has been implemented on a prototype PMSM where the numerical solutions for the magnetic flux linkages in the PMSM are provided. The back-EMF method has been experimentally verified on a prototype PMSM by comparing the estimated Euler angles and angular velocities with measured results of a commercialized gyroscope. As an application, the acquired motion states are used for parameter estimation of the PMSM.

2. Back-EMF for multi-DOF motion sensing

Consider Fig. 1(a) where a closed-loop conductor C (with cross-sectional area of a) surrounded by a magnetic field (denoted with magnetic flux density $\mathbf{B}$). The magnetic flux Φ through S (enclosed by C with surface normal $\mathbf{n}$) is

$$\Phi = \int_S (\mathbf{B} \cdot \mathbf{n}) ds. \tag{1}$$

With the magnetic vector potential $\mathbf{A}$ defined in (2a) and the Stokes' theorem, the surface integral (1) can be reduced to a line integral (2b) where $\boldsymbol{l}$ is the directional vector of C:

$$\nabla \times \mathbf{A} = \mathbf{B}; \tag{2a}$$

$$\Phi = \int_C (\mathbf{A} \cdot \mathbf{l}) dc \tag{2b}$$

For an electro-magnet (EM) which can be treated as a contiguous filamentary conductor (Fig. 1b), the total magnetic flux linkage (MFL) through the EM winding can be obtained by extending (2b) to a volume integral (3) where V is the volume of the EM winding:

$$\Lambda = \frac{1}{a} \int_V (\mathbf{A} \cdot \mathbf{l}) dv \tag{3}$$

For an electromagnetic motion system consisting of both permanent magnets (PMs) and EMs, the MFL in an EM is contributed by the magnetic fields from both the PMs and EMs in the system, therefore

$$\Lambda = \Lambda_{\mathrm{P}} + \Lambda_{\mathrm{E}} \tag{4a}$$

and

$$\mathbf{A} = \mathbf{A}_{\mathrm{P}} + \mathbf{A}_{\mathrm{E}}, \tag{4b}$$

where

$$\Lambda_{\mathrm{P}} = \frac{1}{a} \int_V (\mathbf{A}_{\mathrm{P}} \cdot \mathbf{l}) dv \tag{4c}$$

and

$$\Lambda_{\mathrm{E}} = \frac{1}{a} \int_V (\mathbf{A}_{\mathrm{E}} \cdot \mathbf{l}) dv. \tag{4d)}$$

The equations for computing the magnetic vector potentials of a PM and an EM are given in Appendix A.

The electromotive force (EMF) ε induced in an EM is contributed by two parts; ε_{P} due to the motion of the rotor PMs, and ε_{E} because of changes in current inputs (applied on the self- and mutual-inductances) of the EMs. According to the Faraday's Law of induction [25],

$$\varepsilon = -\frac{d\Lambda}{dt} = \varepsilon_{\mathrm{P}} + \varepsilon_{\mathrm{E}}, \tag{5a}$$

where

$$\varepsilon_{\mathrm{P}} = -\frac{d\Lambda_{\mathrm{P}}}{dt} \tag{5b}$$

and

$$\varepsilon_{\mathrm{E}} = -\frac{d\Lambda_{\mathrm{E}}}{dt}. \tag{5c}$$

For systems with constant inductances, ε_{E} is known when the current inputs through the EMs are specified. This provides a basis for developing an explicit model for a PMSM to characterize the relationship between its multi-DOF rotor motion and the EMFs in the stator EMs; and hence the rotor motion states can be acquired by measuring the voltages in the EMs.

2.1. EMF model in a single EM-PM pair

Fig. 2 shows an EM-PM pair in a PMSM, where both PM and EM are cylindrical; XYZ and xyz (sharing a common origin) are the stator reference and rotor local frames respectively; $\mathbf{s}$ and $\mathbf{r}$ represent the position vectors in XYZ frame from the origin to the geometrical centers of the stator EM and rotor PM; and σ is the separation angle between $\mathbf{s}$ and $\mathbf{r}$. In Fig. 2, y' and x'' are two intermediate axes for defining XYZ Euler angles (α, β, γ). Given the PM position vector $\mathbf{p}$ in xyz coordinates, $\mathbf{r}$ can be obtained from the rotation matrix $[\mathbf{R}]$ in (6a, b) where S and C represent sine and cosine of the Euler angles (subscripts) respectively:

$$\mathbf{r} = [\mathbf{R}]^{\mathrm{T}} \mathbf{p}, \tag{6a}$$

Please cite this article as: K. Bai et al., A magnetic flux model based method for detecting multi-DOF motion of a permanent magnet spherical motor, Mechatronics (2016), http://dx.doi.org/10.1016/j.mechatronics.2016.06.001