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Differentially constrained path planning with graduated state space☆

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ABSTRACT

Complex robotic systems often have to operate in large environments. At the same time, their dynamic is complex enough that path planning algorithms need to reason about the differential constraints of these systems. On the other hand, such robotic systems are typically expected to operate with speed that is commensurate with that of humans. This poses stringent limitation on available planning time. A common approach is to use a two-dimensional(2-D) global planner for long range planning, and a short range higher dimensional planner or controller capable of satisfying all of the constraints on motion. However, this approach is incomplete and can result in oscillations or even the inability to find a path to the goal. In this paper, we present an approach to solve this problem by combining the global and local path planning problem into a single search scheme using a combined 2-D and higher dimensional state space. The proposed approach is demonstrated and validated in simulation and experiment with a significantly asymmetric differential drive robot.

1. Introduction

Mobile robots often have to operate in large complex environments. As such, path planning for these systems needs to account for the various kinodynamic constraints of the platform and is recognized as an important and hard problem in robotics [1]. The search space dimensionality, system dynamics, uncertainty and partial knowledge about the environment further contribute to the difficulty of this problem. The kinodynamic and environmental constraints will result in a high dimensional state space. Unfortunately, this high dimensionality often leads to a dramatic increase in the time and memory cost for finding a path as the environment size increases. Fig. 1 shows the experiment results of searching with 2-D(x, y) state space and 3-D(x, y, θ) state space using *ARA** algorithm [2] respectively. As shown in the figure, the number of expanding states (Fig. 1a) and the planning cost time (Fig. 1b) are increasing largely as the search space dimensionality is changing from 2-D(x, y) to 3-D(x, y, θ).

For a sufficiently large outdoor environment, path planning with kinodynamic constraints can become computationally intractable for the robot's onboard processing capability. Planning only in two dimensions, such as planar position (x, y), does not suffice as the resulting path may not be feasible due to motion constraints or asymmetric platform shapes. As a result, planning in a high dimensional (high-D) state space is often necessary in order to guarantee executable paths. For example, the 2-D (x, y) path shown in Fig. 2a is difficult for a nonholonomic robot to follow due to the instantaneous change in the orientation of the robot. On the contrary, the 3-D(x, y, θ) path shown in Fig. 2b is relatively easy to execute.

As known from Figs. 1 and2, path planning needs to reason about the robot's kinodynamic constraints. On the other hand, such robotic systems are typically expected to operate with speed that is commensurate with that of humans. This poses stringent limitation on available planning time. Therefore, path planning with kinodynamic constraints will result in a contradiction between planning efficiency and the dimensions of the state space determined by the kinodynamic constraints. The main contribution of this paper is to solve this problem. In the next section we discuss the state-of-the-art in path planning with kinodynamic constraints. Section 3 presents the overview of our proposed method. Section 4 provides the method to generate the motion primitives by encapsulating the differential constraints. Section 5 presents the constructed graduated state space which consists of high dimensional state space and low dimensional state space. Section 6 details the combined

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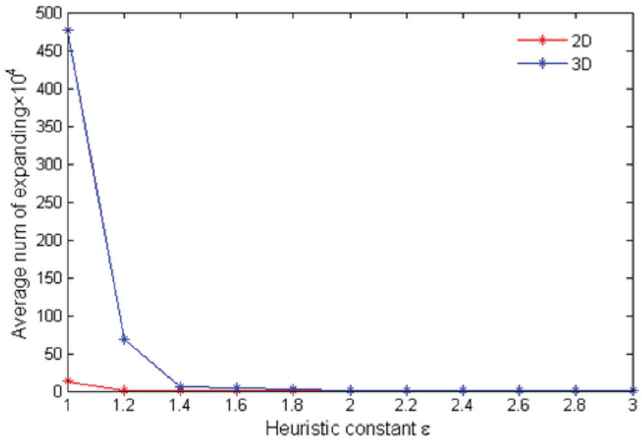
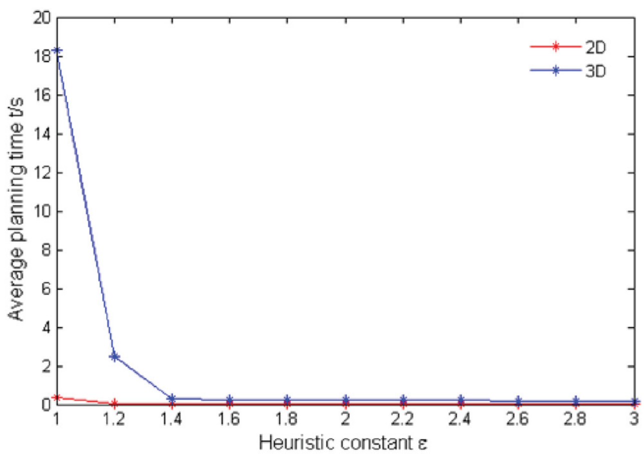


(a) Comparative results of the number of expanded states



(b) Comparative results of path planning time

Fig. 1. The path planning results with 2D(x, y) and 3D(x, y, θ) state space using ARA* algorithm.

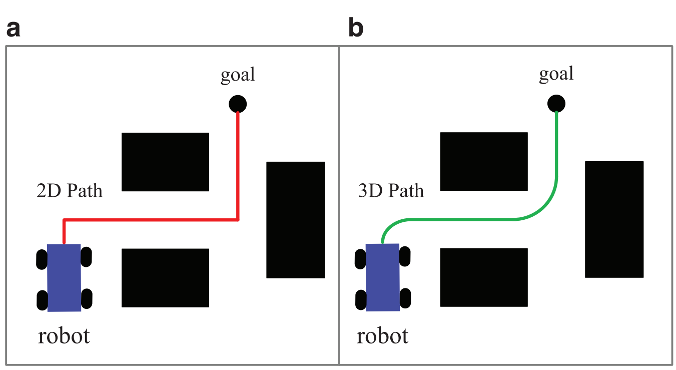


Fig. 2. (a) Unfeasible 2-D path for a nonholonomic robot, (b) feasible 3D- path for a nonholonomic robot.

graph planning algorithm used for path planning. Subsequently, both simulation and real-world robot validated results are presented in Section 7. The paper concludes with lessons learned in Section 8.

2. Related work

Path planning with differential constraints encompasses nonholonomic and kinodynamic planning and is recognized as an important and hard problem in robotics [3–9] . A great variety of

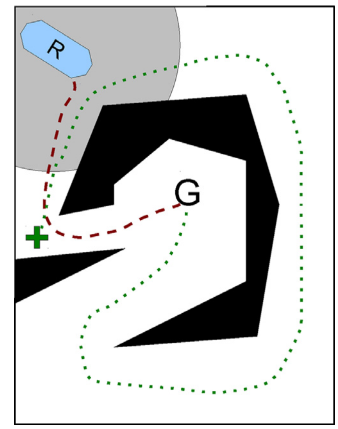


Fig. 3. Example of environment that could be incomplete for systems with separate local and global planners. Robot R is at the start location in the upper left. The local planner operates within the gray circle surrounding the robot. The global planner initially generates red dashed path to the goal G. However the robots size prevents it from making the turn marked by the green "+". At this point the global planner returns the green dotted path. Once the robot returns to the vicinity of the start location, any replan by the global planner would return a path similar to the red dashed path, potentially resulting in the robot oscillating between the two points. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

approaches are being proposed to this day. Differential constraints are important to consider in path planning because many relevant types of robotics platforms impose them. Systems such as wheeled car-like vehicles and tractor-trailer combinations are classical examples of such kinematics constraints [10–12]. Even though different constraints are commonplace in robotics, their degree of severity varies. In order to consider the different constraints, some approaches perform full-dimensional planning for navigation. However, these approaches are either suboptimal [6,13]) or are limited in the size of the area they can handle [14].

For very large environments, many approaches attempt to ignore such constraints because that significantly simplifies path planning. It is often done by relaxing the requirements on the robot (reducing speed, simplifying robot's environment), by pushing the complexity to the robot's hardware, and by attempting to build motion controllers that would eliminate the disturbance caused by ignoring such constraints at planning time. A common alternative is to perform a global plan in 2-D and have a separate local controller or local planner to perform a higher dimensional plan on a small local region around the robot [14–17]. While effectively ignoring a subset of the dimensions for the global plan can make these large environments tractable and reduce planning times, the resulting performance losses are too great in a variety of relevant scenarios. However, some systems cannot afford to ignore their mobility constraints, such as mobile robots with large minimum turning radius [18], tractor-trailer systems [10] and indoor robots pushing carts [19]. Ignoring the differential constraints of such systems will lead to substantially lower productivity than that of these platforms are capable of, due to overly conservative operation. In the worst case, a complete failure, the robot may get stuck and never reach the goal even though there is a feasible path, as the robot controller becomes unable to execute a trajectory that was planned without regard to the system's differential constraints. The main reason for this is the inconsistency between the assumptions that local and global planners make.

As an example, Fig. 3 depicts a scenario that many planners are incapable of successfully dealing with. In this scenario, a short path to the goal is available but due to the sharp turn initially being located outside of the range of the local high dimensional planner, it is infeasible unbeknownst to the robot. As the robot

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